Images and Norms*

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Abstract

An *image conscious* decision maker (DM) cares not only about the physical consequences of his actions, but also about how these actions can be rationalized—are his actions considered generous, patient, sophisticated, etc. Image consciousness nests both shame driven preferences (wanting to conceal information) and signaling (wanting to reveal information; e.g., conspicuous consumption). A specific type of image consciousness is *norm driven behavior* whereby the DM derives additional utility whenever his choices are consistent with a set of prescribed norms. The notion of consistency that is employed determines the DM's attitude toward information revelation. This paper axiomatizes the behavior of an image/norm concerned DM, and identifies the DM value to inducing an image or adhering to a norm.

Key words: Image consciousness; norm consciousness; menu-dependent preferences; reluctant giving.

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1 Introduction

An *image conscious* decision maker (DM) cares not only about the physical consequences of his actions, but also about how these actions can be rationalized—are his actions considered generous, patient, sophisticated, etc. An *image* in this paper is a set of preferences; specifically, when a DM takes an action, the resulting image is the set of preferences that are consistent with his choice. Therefore, a DM's image depends both on his choice and also on what else *could have been* chosen, indicating that an image conscious DM will not behave in accordance to the classical model of choice theory.

Image consciousness nests two distinct phenomena that have been studied in isolation. The first is the DM's desire to unveil information about his motivation and the second his desire to conceal it. By examining these motivations in conjunction, we see that they are in fact captured by the same general framework, and that it is consistent that a DM might conceal information in some situations and divulge it in others.

Example 1. Slothrop is deciding where to take Katje on a date. There are three restaurants, D^l, D^m, and D^h equal in all ways excepting their wine lists. The wine list at restaurant D^l offers only an inexpensive low quality bottle (l); D^m offers this and also a mid-tier bottle (m); D^h, in addition to l and m, offers a costly and high quality bottle (h).

Despite the fact that Slothrop is an absolute cheapskate, he wishes to appear generous and refined. That is, privately, Slothrop would prefer to consume l, but, all else equal, would prefer Katje to think that he prefers more expensive items to less. Hence, at D^m, figuring it worth the small expense to impress Katje, he would publicly choose m. When at D^h, however, Slothrop would revert to his private optimum, choosing l. This is because when m is chosen in favor of h, Katje rules out the possibility that Slothrop prefers grandeur, believing instead that he has middling taste; the cost of sending a signal of refinement—choosing h—is now too high. ∎

The preferences of an image conscious DM will be reflected in his preference over choice problems themselves. As a starting point, I take as the primitive a pair of choice functions: a second stage choice problem defined on sets of consumption alternatives (the choice from a wine list at a given restaurant), and a first stage choice function defined on sets of second stage choice problems (the choice of restaurant). The interpretation is that image concerns are not relevant in the first stage choice problem either because these choices are not observed by the parties judging the DM or simply because these choices are too abstracted from actual consumption for normative issues to arise.[1]

In the example, the addition of m to D^l allows Slothrop to manipulate his image without sacrificing too much in terms of personal consumption value. Further adding h changes the

[1]This latter justification is substantiated by Gino et al. (2016); Grossman and Van Der Weele (2017) who show that "meta-decisions"—which do not implement actions directly, but affect the decision making process; e.g., information collection—and consumption-implementing decisions have different normative implications and different effects on shaping self-images.

image associated with choosing m, causing him to revert his choice. Notice that at both restaurants, he could choose l, effecting the same image (that he is a cheapskate) and the same physical consumption. The fact that he did not choose l from D^m signals that he must prefer his induced outcome, and so, from the ex-ante perspective, he must prefer D^m to D^h.

This intuition is captured by *Image Consistency*, an axiom tying together the two stages of choice. Let D and D' be two second stage choice problems containing x and such that choosing x induces the same image, I, from either choice problem. *Image Consistency* states that if x is chosen from D' in the second stage, then D must be weakly preferred to D' in the first stage.

Norm Driven Behavior. A particular type of image consciousness is *norm driven* behavior. A *norm*, like an image, is a set of preferences—for example, social preferences satisfying some level of altruism. A DM adheres to the norm when his actions can be rationalized by normative choices: when the image associated with his action is consistent with the norm. Just as in the more general image conscious model, normative concerns can drive the DM to both unveil or conceal information: this distinction depends completely on how we define an image being "consistent" with a norm.

A norm is *active* if the DM adheres to the norm whenever his actions can only be rationalized by normative behavior (that is, every preference in the image is a normative preference) and *passive* if he adheres whenever his actions can be rationalized by some normative behavior (that is, there exists a preference in the image that is a normative preference). For an active norm, the burden of proof lies on the DM, his choice must prove his consistency with the norm; with a passive norm the DM is 'innocent unless proven guilty,' to adhere he simply must not exhibit inconsistency.

Under active norms the DM has a preference to reveal information in the sense that he prefers, all else equal, to induce smaller, more informative images to larger ones. This is because in order to adhere to a norm, the DM must activly show consistency, which is easier the smaller the image. The opposite logic insists that under passive norms, the DM prefers to induce larger images and so prefers to conceal information.

Example 2. Tchitcherine chooses between payoff vectors for himself and for his girlfriend Geli: consumption objects are points in $\mathbb{R}^2$. Tchitcherine cares about both payoffs, but cares weakly more about his own. The set of such utilities can be normalized to be of the form $x_t + \beta x_g$, with $\beta \in [0, 1]$. The two extremes represent purely selfish and purely utilitarian preferences.

An image, I, in this model can be thought of as an interval of possible β's. Given the collection of consumption objects $D \subset \mathbb{R}^2$, the image induced by choosing $x \in D$ is $I^x_D = \{\beta \in [0, 1] \mid x_t + \beta x_g \geq y_t + \beta y_g, \forall y \in D\}$. If Tchitcherine is image conscious his total utility from choosing $x \in D$ is

$$x_t + \beta x_g + \Gamma(I_D^x),$$

for some function Γ.

Specifically, Tchitcherine, independently of how much he actually cares about Geli's payoff (his true β), prefers to adhere to the altruistic norm where $\beta \geq \frac{1}{2}$. If the norm is active, then Tchitcherine is considered selfish unless he explicitly shows otherwise, in which case he derives additional utility for adhering to the norm. Conversely, if the norm is passive, Tchitcherine is considered altruistic, deriving additional utility, unless he explicitly demonstrates otherwise. These cases are instantiated by

$$\Gamma^A(I) = \begin{cases} 1 \text{ if } I \subseteq [\frac{1}{2}, 1] \\ 0 \text{ otherwise} \end{cases} \qquad \text{and} \qquad \Gamma^P(I) = \begin{cases} 1 \text{ if } I \cap [\frac{1}{2}, 1] \neq \emptyset \\ 0 \text{ otherwise,} \end{cases}$$

respectively. The passive norm model is a special case of Dillenberger and Sadowski (2012).
$\blacksquare$

Since making a choice is informative exactly because it indicates that the chosen action was preferred the unchosen actions, larger choice problems are more informative than smaller ones. Thus, under active norms, where the DM must actively reveal adherence to the norm, the DM will prefer larger choice problems and display a Krepsian preference for flexibility (Kreps, 1979). Conversely, under passive norms, the DM will prefer, ceteris paribus, smaller menus to larger, and display upper-set-betweenness. Thus, image/norm consciousness provides a new interpretation of well known axioms regarding menu preferences.

Image Conciousness without First Stage Choices. The model considered below assumes the modeler can observe two stages of choice. However, if first stage choices are *completely* private, they are by definition unobservable to the modeler. Nonetheless, recall that choice reversals indicate a preference for one image over another, an inference that relies only on second stage choices.

In Appendix B, I consider a variant of the model where only the second stage choice is observable, and provide axioms to ensure a representation equivalent to the two-stage model. But, uniqueness is no longer possible—the parameters of the DM's image concerns are no longer identifiable. Thus, while the 2-stage primitive of the general model is indeed demanding, the results of Appendix B can be seen as a demonstration that it is necessary.[2]

Organization. The next section outlines the requisite notation and introduces the IC representation. Then, Section 3 provides the axiomatization and representation theorem

[2] This is a specific symptom of a larger problem: if the first stage choices were observable but the DM entertained normative concerns there also, again identification would be lost (unless the modeler could observe image-concern-free 0^{th} stage choices over sets of stage 1 choice problems). Indeed, unless the model is closed in some way, identification is not possible. Perhaps some kind of inverse limit construction (for example IHCPs in Gul and Pesendorfer (2004)) would allow for closure/identification without exogenously imposing a stage at which image concerns do not apply.

under the assumption that both stages of choice are observable (this assumption is relaxed in Appendix B, which provides a representation when only the second stage is observed). Section 4 introduces a subclass of IC representations that are norm-driven and explores the relationship between active/passive norms and the DM's attitude towards information revelation. Simple comparative statics exercises are found in Section 5. Literature is reviewed at the end, in Section 6.

2 Notation and Primitives

Notation. Consumption takes place in $\mathbb{R}^n$ for $n \geq 2$. This generality permits the instantiation that consumption objects are lotteries, social allocations regarding multiple agents, Anscombe-Aumann acts, sequences of consumption over time, etc. Let $\mathcal{D}$ denote the set finite non-empty subsets of $\mathbb{R}^n$, referred to as stage 2 choice problems (2CPs). The topology on $\mathcal{D}$ is induced by the Hausdorff metric, $d_h^{\mathcal{D}}$. Let $\mathscr{M}$ denote the set of all finite non-empty subsets of $\mathcal{D}$, refereed to as stage 1 choice problems (1CPs). Endow $\mathscr{M}$ with the associated Hausdorff metric, $d_h^{\mathscr{M}}$. For any $k \in \mathbb{N}$ let $\mathcal{D}^k = \{D \in \mathcal{D} \mid \#[D] = k\}$ denote the set of 2CPs with n elements.

Our primitive is a pair $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$, where $\mathscr{C}_1$ is a choice function over $\mathscr{M}$ (i.e., $\mathscr{C}_1 : \mathscr{M} \to \mathscr{M}$ such that $\mathscr{C}_1(\mathcal{M}) \subseteq \mathcal{M}$ for all $\mathcal{M} \in \mathscr{M}$) and $\mathcal{C}_2$ is a choice function over $\mathcal{D}$ (i.e., $\mathcal{C}_2 : \mathcal{D} \to \mathcal{D}$ such that $\mathscr{C}_1(D) \subseteq D$ for all $D \in \mathcal{D}$). The interpretation is that of two stage choice; in the first stage the DM faces a 1CP, $\mathcal{M}$, (a set of 2CPs) from which to choose the next periods constraint. His first stage choice is given by $\mathscr{C}_1(\mathcal{M})$. In the next period he faces one of the acceptable 1CPs $D \in \mathscr{C}_1(\mathcal{M})$, from which he must make a choice of consumption object. His second stage choice is given by $\mathcal{C}_2(D)$.[3]

For sets $D, D' \in \mathcal{D}$ and $\lambda, \lambda' \in \mathbb{R}$, define $\lambda D + \lambda' D' = \{\lambda x + \lambda' x' \mid x \in D, x' \in D'\}$. Likewise, for sets $\mathcal{M}, \mathcal{M}' \in \mathscr{M}$ and $\lambda, \lambda' \in \mathbb{R}$, define $\lambda \mathcal{M} + \lambda' \mathcal{M}' = \{\lambda D + \lambda' D' \mid D \in \mathcal{M}, D' \in \mathcal{M}'\}$. For any $D \in \mathcal{D}$ denote by $UC(D)$ the upper contour set of D with respect to $\mathscr{C}_1$: $UC(D) = \{D' \in \mathcal{D} \mid D' \in \mathscr{C}_1(\{D, D'\})\}$, the set of all decision problems chosen over x. Define the lower contour set, $LC(D)$, in dual fashion $LC(D) = \{D' \mid D \in UC(D')\}$.

Utilities and Representation. For each $u \in \mathbb{R}^n$, u defines a linear representation (i.e., expected utility function) over $\mathbb{R}^n$. This is via the obvious duality which views u as the function taking x to its inner product with u. When a decision maker makes a public choice, he receives utility directly from consumption, but also, from his *image*—the set of utilities that observers believe he might have. Of course, choices over lotteries can only reveal utilities up to affine transformations, so we identify utilities which are rescalings of one another: call $I \subset \mathbb{R}^n$ an *image* if I is closed, convex and $\lambda I \subseteq I$ for all $\lambda > 0$. Let $\mathbb{I}$ denote the set of all images.

[3] In general, flat font face is used for consumption objects (i.e., x, D), calligraphic lettering is used second stage choice objects (i.e, $\mathcal{M}, \mathcal{D}, \mathcal{C}_2$) and script lettering is used for first stage choice objects (i.e. $\mathscr{M}$, $\mathscr{C}_1$).

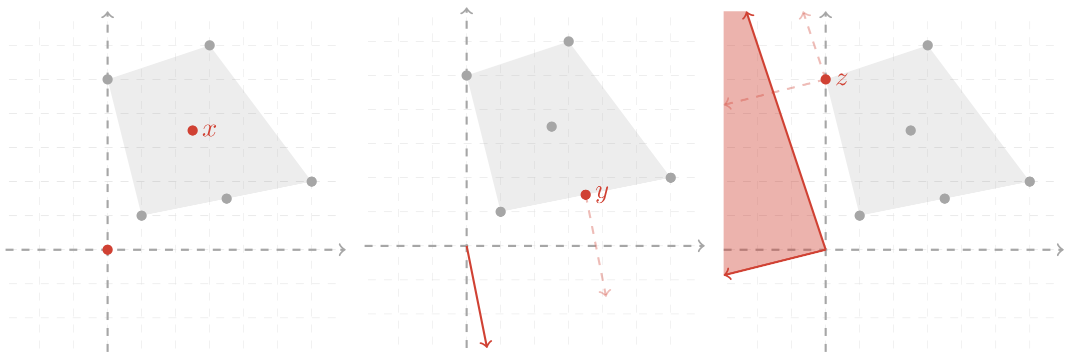

Figure 1: I^w_D for $w \in \{x, y, z\}$ and the same D. The image is shown in red. The dotted lines, representing the image shifted to the chosen object, highlight the geometric dependence between choice problems and images.

Given any $D \in \mathcal{D}$ and $x \in D$ we denote by $I^x_D \in \mathbb{I}$ the set of utilities such that x would maximize u given D. We have

$$I^x_D = \{u \in \mathbb{R}^n \mid u(x) \geq u(y), \text{ for all } y \in D\}.$$

Notice that I^x_D depends only on the convex hull of D, so we can view it as an operation on convex sets, where it is referred to in the general literature as the *normal cone* of x in D. Given $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ let $\textsc{img}(\mathcal{C}_2)$ denote the set $\{I \in \mathbb{I} \mid \exists (x, D), x \in \mathcal{C}_2(D), I^x_D = I\}$ referred to as the *realized images.*

With these definitions in place, we can formally state the definition of an image conscious representation.

Definition. An *image conscious representation* of $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ is a pair $\langle u, \Gamma \rangle$ where $u \in \mathbb{R}^n$ and $\Gamma : \mathbb{I} \to \mathbb{R} \cup \{-\infty\}$, such that

$$\mathcal{C}_2(D) = \operatorname*{arg\,max}_{x \in D} \big(u(x) + \Gamma(I^x_D)\big) \qquad \text{and} \tag{C2}$$

$$\mathscr{C}_1(\mathcal{M}) = \operatorname*{arg\,max}_{D \in \mathcal{M}} \Big(\max_{x \in D} \big(u(x) + \Gamma(I^x_D)\big)\Big), \tag{C1}$$

for all $D \in \mathcal{D}$ and $\mathcal{M} \in \mathscr{M}$, and $\Gamma(I) > -\infty$ if and only if $I \in \textsc{img}(\mathcal{C}_2)$.

3 Two Stage Image Conscious Choice

Because the DM makes the first stage privately, he has a more standard preference in the first stage. Specifically, there are no context effects, and so the DM will have a well defined value function over decision problems. When considering the class of degenerate (i.e., singleton) choice problems, every second stage choice, by nature of being degenerate, instills the same image. Hence the DM's consideration of such problems depends only on his consumption utility.

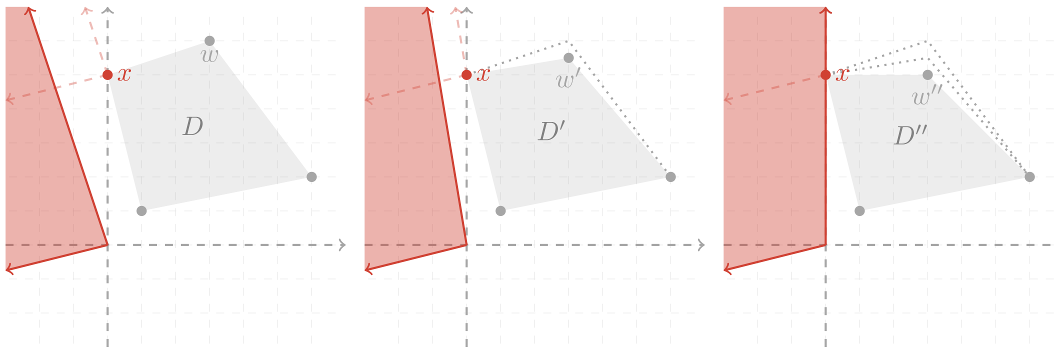

Figure 2: When the element w is replaced with w', then w'' the image associated with choosing x expands. Under each replacement, the new choice problem lies within the convex hull of the previous ones. Hence: $I_D^x \subset I_{D'}^x \subset I_{D''}^x$

Axiom 1—Singleton Expected Utility. There exists a value function, $V : \mathcal{D} \to \mathbb{R}$ such that

$$\mathscr{C}_1(\mathcal{M}) = \big\{ D \in \mathcal{M} \mid D \in \arg\max_{\mathcal{M}} V(D) \big\},$$

where $u : x \mapsto V(\{x\})$ is a linear function over $\mathbb{R}^n$.

A1 imposes a lot of structure and is admittedly somewhat divorced from the observable primitive. Because of this, Appendix A contains a (rather straightforward if uninspiring) set of axioms directly on the primitive, equivalent to A1.

The next axiom ensures that an image depends only on the difference between what was chosen and what could have been, and not the aggregate level of consumption. In other words, changing the baseline level of consumption by shifting all consumption alternatives by a constant amount does not distort the image concerns.

Axiom 2—Translation Invariance. For all $x \in \mathbb{R}^n$, $\mathcal{M} \in \mathscr{M}$ and $D \in \mathcal{D}$,

$$\mathscr{C}_1(\mathcal{M} + \{\{x\}\}) = \mathscr{C}_1(\mathcal{M}) + \{\{x\}\} \text{ and}$$
$$\mathcal{C}_2(D + \{x\}) = \mathcal{C}_2(D) + \{x\}$$

What remains is to ensure the value function V reflects image consciousness. In particular, we want to show that the gap between $V(D)$ and $u(\mathcal{C}_2(D))$ is dependent only on the induced image.

When $I_D^x = I_{D'}^x$, then choosing x from D or D' induces the same perception in an observer; hence, the DM should find choosing x from D exactly as appealing as choosing x from D'. Thus, if x is chosen from D but not chosen from D' it must mean that whatever is chosen from D' is even better than consuming x with image I_D^x. From a period 1 perspective, this indicates that D' is preferred to D. The following axiom embodies exactly this logic.

Axiom 3—Image Consistency. Assume $I_D^x = I_{D'}^x$ and $x \in \mathcal{C}_2(D)$. Then $x \in \mathcal{C}_2(D')$ if and only if $D \in \mathscr{C}_1(\{D, D'\})$

Image consistency, along with the linear structure imposed by A1 and A2, is enough to ensure the existence of an image conscious representation.

3.1 Representation Result

Theorem 3.1. *The following are equivalent:*

1. $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ *satisfies* A1–3,

2. $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ *satisfies* A2–5,

3. $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ *has an image conscious representation* $\langle u, \Gamma \rangle$.

Moreover, u *is unique up to positive linear translations, and* Γ *is unique up to an additive constant (on its effective domain).*

Proof. The equivalence between (1) and (2) is given by Lemma 1. That (3) implies (1) is standard. We will show that (1) implies (3). Let $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ satisfy A1 and A3. Take u and V as defined in A1. For each $I \in \mathbb{I}$, define $\Gamma(I)$ to be

$$\Gamma(I) = \begin{cases} V(D) - u(x) & \text{if there exists } (D, x) \text{ with } x \in \mathcal{C}_2(D) \text{ and } I_D^x = I \\ -\infty & \text{otherwise.} \end{cases} \tag{3.1}$$

We here verify that Γ is well defined. Let (D, x) and (D', x') be such that $I_D^x = I_{D'}^{x'}$ and $x \in \mathcal{C}_2(D)$ and $x' \in \mathcal{C}_2(D')$. By A2 it is without loss of generality to assume $x = x'$. By A3 it must be that $V(D) = V(D')$. This of course implies that $V(D) - u(x) = V(D') - u(x)$, so that Γ is well defined.

Next, we claim that for all D we have $\mathcal{C}_2(D) = \arg\max_{x \in D} u(x) - \Gamma(I_D^x)$. First, assume that $x \in \mathcal{C}_2(D)$ and let $y \in D$. Since $x \in \mathcal{C}_2(D)$ it follows that $\Gamma(I_D^x) \neq -\infty$, so if $\Gamma(I_D^y) = -\infty$, we have $u(x) + \Gamma(I_D^x) \geq u(y) + \Gamma(I_D^y)$ immediately. So, to make the problem extra hard, assume $\Gamma(I_D^y) \neq -\infty$. It must be there exists a (D', y') such that $I_{D'}^{y'} = I_D^y$ with $y' \in \mathcal{C}_2(D')$. By A2, we can choose $y' = y$, so that $I_{D'}^y = I_D^y$ and that $y \in \mathcal{C}_2(D')$. Thus, we can conclude that $V(D) \geq V(D')$. Appealing to (3.1) implies that

$$u(x) + \Gamma(I_D^x) = V(D) \geq V(D') = u(y) + \Gamma(I_{D'}^y) = u(y) + \Gamma(I_D^y).$$

Next assume that $x \in \arg\max_{x \in D} u(x) - \Gamma(I_D^x)$ but $x \notin \mathcal{C}_2(D)$. Let $y \in \mathcal{C}_2(D)$. This implies $\Gamma(I_D^y) \neq -\infty$, and so also that $\Gamma(I_D^x) \neq -\infty$. Thus, there exists a (D', x) such that $I_{D'}^x = I_D^x$ with $x \in \mathcal{C}_2(D')$; since $x \in \mathcal{C}_2(D')$ and $x \notin \mathcal{C}_2(D)$ we can conclude via A3 that $V(D) > V(D')$. Hence, by the definition of Γ,

$$u(y) + \Gamma(I_D^y) > u(x) + \Gamma(I_{D'}^x) = u(x) + \Gamma(I_D^x),$$

another clear contradiction.

∎

3.2 Attitudes towards information revelation

A DM who has a *preference for revelation* wants to reveal the motivations behind his actions, providing the observer with the maximal information about his preference. For example, a generous, if vain, DM who wishes to make it known that he is generous. Conversely, a DM who has a *preference for concealment* prefers, all else equal, to limit the inference an observer can make regarding his underlying motivations. For example, the selfish DM who wishes to take selfish actions without tarnishing his reputation. Formally:

Definition. A DM has a *preference for revelation* if for $I, J \in \mathbb{I}$, $I \subseteq J$ implies that $\Gamma(J) \leq \Gamma(I)$, and has a *preference for concealment* if $I \subseteq J$ implies that $\Gamma(I) \leq \Gamma(J)$.

There is an interesting link between attitudes towards the revelation of information and classical axioms placed on menu preferences. In particular, set betweenness, which plays a key role in Dillenberger and Sadowski (2012), arises from a preference for concealing information, whereas Krepsian flexibility from a preference for revealing information.

Theorem 3.2. *Assume that $\langle u, \Gamma \rangle$ represents $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$, then if a DM has a preference for revelation, he displays a preference for flexibility: $D \cup D' \in \mathscr{C}_1\{D, D \cup D'\}$. Conversely, he has a preference concealment then he displays upper set betweenness: $D \in \mathscr{C}_1\{D, D'\}$ implies $D \in \mathscr{C}_1\{D, D \cup D'\}$.*

Proof. Assume that $\langle u, \Gamma \rangle$ represents $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$. Assume the DM has a preference for revelation. Notice that for all $D, D' \in \mathcal{D}$ and $x \in D$, $I^x_{D \cup D'} \subseteq I^x_D$, hence, by the definition of a preference for revelation,

$$V(D \cup D') \geq \max_{x \in D} u(x) + \Gamma(I^x_{D \cup D'}) \geq \max_{x \in D} u(x) + \Gamma(I^x_D) = V(D).$$

The proof for concealment is parallel and so omitted. ∎

If a DM (globally) displays both a preference for concealment and revelation, then every image is valued equally and the DM adheres to strategic rationality, so that each menu is indifferent to its maximizer. However, as evidenced by Example 1, these attitudes towards information are generally not held globally; the DM sometimes (strictly) wants to reveal information about his motivation, and sometimes (strictly) wants to conceal it. In the example, Slothrop values the addition of a medium priced wine because it allows him to send a signal placing him in a favorable light. He dislikes the further addition of a high priced wine, though, since its existence places bounds on his taste for quality should he continue to choose the medium quality wine. As such, he prefers $\{l, m\}$ to both $\{m\}$ and $\{l, m, h\}$—a violation of both global preferences to information.

4 Norm Driven Behavior

An important special case of image consciousness is normative decision making. A *norm*, $N \subset \mathbb{R}^n$, is a set of utilities that the DM feels as if he should adhere to; the image utility derived from $I \in \mathbb{I}$ is determined by the relation between I and N. A norm, N, is *active* if the DM adheres to the norm whenever $I \subseteq N$ and *passive* if he adheres whenever $I \cap N \neq \emptyset$. For an active norm, the burden of proof lies on the DM, his choice must prove his consistency with the norm; with a passive norm the DM is 'innocent unless proven guilty,' to adhere he simply must not exhibit inconsistency.

A DM is *normative* if there exists a set of norms, nested by set inclusion, such that the DM's image utility is determined by the smallest norm he adheres to. Formally:

Definition. Call $\langle u, \Gamma \rangle$ *actively normative* if there exists a set $\{N_r\}_{r \in \mathbb{R}}$ such that (i) $N_r \subseteq \mathbb{R}^n$ for each r, (ii) $r \leq r'$ implies $N_r \supseteq N_{r'}$ and (iii) for all $I \in \text{IMG}(\mathcal{C}_2)$,

$$\Gamma(I) = \sup\{r \mid I \subseteq N_r\}. \tag{AN}$$

Call $\langle u, \Gamma \rangle$ *passively normative* if condition (iii) is replaced by (iii') for all $I \in \text{IMG}(\mathcal{C}_2)$,

$$\Gamma(I) = \sup\{r \mid I \cap N_r \neq \emptyset\}. \tag{PN}$$

To better understand this general definition of normative preferences, and how normative concerns can give rise to a preference for revelation/concealment, consider the following variation of Example 2.

Example 3. Again, Tchitcherine's utility for choosing x from D is given by

$$x_t + \beta x_g + \Gamma(I_D^x),$$

where I_D^x is associated with a closed subinterval of $[0, 1]$.

Consider the set of active norms given by $N_r = [r, \infty)$, so that $\Gamma(I) = \sup\{r \mid I \subset [r, \infty)\} = \min_{r \in I} r$ so that the image utility of I is the lower bound of the associated interval. From this vantage, and the triviality that $I \subset J$ implies $\min_{r \in J} r \leq \min_{r \in I} r$, we can see the DM has a preference for revealing information (i.e., prefers smaller images to larger), since the DM must actively display adherence to a norm. Of course, the converse logic applies to the 'upper-bound' model with norms given by $N_r = (-\infty, 1 - r]$, and where the DM displays a preference for larger images to smaller. Here, the DM is only considered selfish when he explicitly acts selfishly. ∎

Under the above axioms, a DM's choices induce a weak overing $\text{IMG}(\mathcal{C}_2)$ via Γ. Of course this ordering can be directly inferred from the primitives via $\Gamma(I) \geq \Gamma(J)$ if and only if for all D, D' such that $x \in \mathcal{C}_2(D) \cap \mathcal{C}_2(D')$ and $I_D^x = I$ and $I_{D'}^x = J$ we have also that $D \in \mathscr{C}_1(\{D, D'\})$. For the remainder of the section, we will simply refer directly to this ordering on $\text{IMG}(\mathcal{C}_2)$. This is without loss of generality, but simplifies notation considerably.

Theorem 4.1. *(i) A DM is actively normative if and only if for all $\mathbb{J} \subseteq \mathbb{I}$ and $I \in \mathbb{I}$ we have: if $\Gamma(J) \geq \Gamma(I)$ for all $J \in \mathbb{J}$ then $K \subseteq \bigcup_{\mathbb{J}} J$ implies $\Gamma(K) \geq \Gamma(I)$. (ii) A DM is passively normative if and only if for all $\mathbb{J} \subseteq \mathbb{I}$ and $I \in \mathbb{I}$ we have: if $\Gamma(J) \geq \Gamma(I)$ for all $J \in \mathbb{J}$ then $K \supseteq \bigcap_{\mathbb{J}} J$ implies $\Gamma(K) \geq \Gamma(I)$.*

Proof. (of (i), where (ii) is analogous). Necessity is obvious. Towards sufficiency, assume the latter condition. For each $r \in \mathbb{R}$ set

$$N_r = \bigcup\{J \in \mathbb{I} \mid \Gamma(J) \geq r\}.$$

We claim this set of norms represents Γ via (AN). Take some $I \in \text{IMG}(\mathcal{C}_2)$ with $\Gamma(I) = r$. Thus, $I \in \{J \in \mathbb{I} \mid \Gamma(J) \geq r\}$, so $I \subseteq N_r$ implying $\sup\{r' \mid J \subseteq N_{r'}\} \geq r$. By way of contradiction assume this latter inequality was strict. Thus, there is some $r' > r$ such that $I \subseteq N_{r'} = \bigcup\{J \in \mathbb{I} \mid \Gamma(J) \geq r'\}$. But, by the condition of the theorem, this implies that $\Gamma(I) \geq r' > r$, a contradiction. ∎

This characterization makes immediate the connection between normative concerns and attitudes towards information revelation.

Corollary 4.2. *(i) If a DM is actively normative he has preference for revelation. (ii) If a DM is passively normative he has preference for concealment.*

Proof. Assume $\langle u, \Gamma\rangle$ is actively normative. Let $I, J \in I$ with $I \subseteq J$. By Theorem 4.1, letting $\mathbb{J} = \{J\}$, we have $\Gamma(I) \geq \Gamma(J)$, as desired. ∎

In some situations, the set of norms may not be linearly ordered by set inclusion. For example, perhaps the norm of acting in a conservative manner and that of acting in a liberal manner are equally valued, but it is undesirable to not take a stance and be a centrist.

Example 4. The images associated with a candidate's choice of political platform is associated with an subinterval of $[0, 1]$, representing the conservative/liberal spectrum. The electorate is divided and positioned at the extremes. Voters at either extreme value the image via its worst case. The value to the candidate is therefore:

$$\Gamma(I) = \max\{\min_{r\in I} r,\ \min_{r\in I} 1 - r\}$$

∎

It may, therefore, initially seem fruitful to consider a weaker notion of normativity where the set of norms, rather than being indexed by the linearly ordered $\mathbb{R}$, is indexed by an arbitrary partially ordered set.

Definition. Call $\langle u, \Gamma\rangle$ *weakly actively normative* if there exists a partially ordered set $(P, \leq)$ and a set $\{N_p\}_{p\in P}$ such that (i) $N_p \subseteq \mathbb{R}^n$ for each r, (ii) $p \leq p'$ implies $N_p \supseteq N_{p'}$ and (iii) for all $I \in \text{IMG}(\mathcal{C}_2)$,

$$\Gamma(I) = \sup\{\varphi(p) \mid I \subseteq N_p\}. \tag{WAN}$$

for some monotone $\varphi : p \to \mathbb{R}$.

Upon closer inspection, however, this definition is simply a convoluted restatement of a preference for revelation (necessity is obvious, and for sufficiency we can take as our partially ordered set $\text{IMG}(\mathcal{C}_2)$ under (the opposite of) set inclusion with $\varphi : I \mapsto \Gamma(I)$). Obviously, the analogous definitions and equivalences will hold for passive norms. This serves as another view to Corollary 4.2.

5 Comparative Image Consciousness

Consider two different IC DMs: $\langle \mathscr{C}_1^i, \mathcal{C}_2^i \rangle$ and $\langle \mathscr{C}_1^j, \mathcal{C}_2^j \rangle$. We are interested in understanding how the image concerns of the two decision makers relate to one another.

Definition. Say that i and j have the *same image ranking* if (i) $\textsc{img}(\mathcal{C}_2^i) = \textsc{img}(\mathcal{C}_2^j)$ (ii) for all $D_i, D_i', D_j, D_j' \in \mathcal{D}$, such that $I_{D_i}^x = I_{D_j}^x$ and $I_{D_i'}^x = I_{D_j'}^x$ and x is chosen from all four 2CPs, then

$$D_i \in \mathscr{C}_1^i(\{D_i, D_i'\}) \iff D_j \in \mathscr{C}_1^j(\{D_j, D_j'\}).$$

When i chooses x from D_i, it imparts the same image as when j choses x from D_j, likewise, when x is chosen from D_i' and D_j'. Since the consumption utility is invariant across the two choices, each DM's ranking of the primed and un-primed 2CPs depends only on his ranking of the induced images. Since the rankings coincide, the DMs rank images identically. The first part of the definition ensures that every pair of images that can be compared in this manner by i can also be compared by j.

Theorem 5.1. *$\langle u^i, \Gamma^i \rangle$ and $\langle u^j, \Gamma^j \rangle$ represent preferences which have the same image ranking if and only if Γ^i is an strictly increasing transformation of Γ^j.*

Proof. Assume i and j have the same image rankings and let $\Gamma^i(I) \geq \Gamma^i(I') > -\infty$. Then there exists some $(x_i, D_i), (x_i', D_i')$ such that $x_i \in \mathcal{C}_2^i(D_i)$, $x_i' \in \mathcal{C}_2^i(D_i')$ and $I_{D_i}^{x_i} = I$ and $I_{D_i'}^{x_i'} = I'$. Since i and j have the same realized images there also exists some analogous $(x_j, D_j), (x_j', D_j')$. By translation invariance, we can choose $x_i = x_i' = x_j = x_j'$. Since consumption utility is constant for each DM, we have

$$\begin{aligned} \Gamma^i(I) \geq \Gamma^j(I') &\iff V^i(D_i) \geq V^i(D_i') \\ &\iff V^j(D_j) \geq V^j(D_j') \iff \Gamma^j(I) \geq \Gamma^j(I'). \end{aligned}$$

The other direction is immediate. ∎

With this definition in place, we can now discuss when DM i is more or less sensitive to image effects than j. Of course, such a definition only has real bite when the DMs also entertain the same image ranking, and the same ranking of consumption alternatives.

Definition. Say that i is *more image conscious* than j if (i) i and for j have the same image ranking, and (ii) $\mathscr{C}_1^i$ and $\mathscr{C}_1^j$ coincide on $\mathcal{D}^1$ and (iii) if $\{y\} \in \mathscr{C}_1^i(\{\{x\},\{y\}\}) \cap \mathscr{C}_1^j(\{\{x\},\{y\}\})$ and $D_i, D_i', D_j, D_j' \in \mathcal{D}$ are such that $I_{D_i}^x = I_{D_j}^x$ and $I_{D_i'}^y = I_{D_j'}^y$ and $x \in \mathcal{C}_2^i(D_i) \cap \mathcal{C}_2^j(D_j)$ and $y \in \mathcal{C}_2^i(D_i') \cap \mathcal{C}_2^j(D_j')$, then

$$D_j \in \mathscr{C}_1^j(\{D_j, D_j'\}) \implies D_i \in \mathscr{C}_1^i(\{D_i, D_i'\}).$$

If i is more image conscious than j, then he cares relatively more about changes in image utility than does j. In the definition, since y is preferred to x, the fact that D_j is chosen by j—so x is consumed—indicates that he finds the image utility more than makes up for gap in consumption utility. Since i must also make the same choices, it must be that he also finds the image utility sufficient to overcome the gap in consumption utility. As expected, this increased sensitivity can be captured by the relation that Γ^i is "more spread out" than Γ^j.

Theorem 5.2. *Let $\langle u^i, \Gamma^i \rangle$ and $\langle u^j, \Gamma^j \rangle$ represent preferences such that i is more image conscious than j then $u^i = u^j$ and*

$$|\Gamma^i(I) - \Gamma^i(I')| > |\Gamma^j(I) - \Gamma^j(I')|$$

for all $I, I' \in \mathbb{I}$.

Proof. Let i be more image conscious than j. That $u^i = u^j$ is immediate, so call the joint representation u. Choose some realized images $I, I' \in \mathbb{I}$ and without loss of generality, assume that $\Gamma^j(I) \geq \Gamma^j(I')$. Choose some $x, y \in \mathbb{R}^n$ such that $\Gamma^j(I) - \Gamma^j(I') = u(y) - u(x) \geq 0$. Then by translation invariance we can find some $D_i, D'_i, D_j, D'_j \in \mathcal{D}$ such that $I^x_{D_i} = I^x_{D_j} = I$ and $I^y_{D'_i} = I^y_{D'_j} = I'$ and $x \in \mathcal{C}^i_2(D_i) \cap \mathcal{C}^j_2(D_j)$ and $y \in \mathcal{C}^i_2(D'_i) \cap \mathcal{C}^j_2(D'_j)$.

By our assumption we have $V^j(D_j) = u(x) + \Gamma^j(I) = u(y) + \Gamma^j(I') = V(D'_j)$. Therefore, since i is more image conscious than j, we have also that $u(x) + \Gamma^i(I) = V^i(D_i) \geq V^i(D'_i) = u(y) + \Gamma^i(I')$. Rearranging yields, $\Gamma^i(I) - \Gamma^i(I') \geq u(y) - u(x) = \Gamma^j(I) - \Gamma^j(I')$, as desired. ∎

6 Discussion, Including a Brief Review of Related Literature

Image conscious behavior is ubiquitous and has long been studied within economics. In a work of classical importance, Veblen (1899) coined the term *conspicuous consumption* referring to purchases in which the primary value is derived indirectly by signaling wealth or status. Such spending habits are alive and well in the modern era.

Recently, experimental economists and psychologists have exposed the importance of image concerns in sundry other contexts. A common theme is the discord between and individual's personal preference and his desire to be seen as acting in a normative manner: the DM faces a tradeoff between direct utility and utility derived from adhering to a norm. This tradeoff is central to the present model as captured by the IC representation and in particular the model of norm driven decision making.

Dana et al. (2006) find that subjects in the dictator game are willing to pay a positive cost to ensure the receivers did not know the game was to be played (i.e., the dictator gets the full pie, less the cost, and the receiver is never informed there was a decision to be made). Because the decision to keep everything is always available, paying the cost serves only to effect a more desirable image. This result is echoed in Andreoni and Bernheim (2009), where subjects' choice of fair (i.e., 50-50) allocations in the dictator game depends very much on who can observer the dictators' choices. When there is a commonly known chance that unfair allocations get implemented irrespective of the dictator's choice, and these nature-chosen outcomes are indistinguishable from dictator-chosen outcomes to receivers, then the rate of fair allocations dramatically declines.

DellaVigna et al. (2012) find, in a door-to-door field experiment, that many donation decisions seem to be predicated on social pressure. When given the ability to avoid face to face contact with a solicitor, donations decrease. This effect is concentrated in small donations, an effect that is predicted by the present model. DellaVigna et al. (2016) find that social pressure plays a key role in the decision to vote; potential voters are more likely to vote when they expect that other will ask them about their voting record.

Bénabou and Tirole (2006) provide the canonical utility function for image concerned agents and explore how direct incentives to act pro-socially can have the opposite effect by skewing the images associated with certain actions. Their model is behavioral rather than decision theoretic, in the sense that they are less concerned with identification from observables, and the generality of the types of images that can be entertained. For instance, they make the assumption that actions can be linearly ordered and that everyone prefers a "higher" image to a "lower" one.

Closest to this model are the models of Dillenberger and Sadowski (2012) and Evren and Minardi (2017) who investigate the axiomatic characterizations of shame driven preferences and of *warm glow*, respectively. Although image consciousness, shame, and warm glow are all distinct phenomena, there are two major similarities between the models: (i) all can promote normative behavior and (ii) in these models, the *menu* from which an element is chosen changes the derived value from consumption.

The Dual-Self Interpretation of Images. It need not be that an IC DM cares about the opinion of any *third* party, but rather, the 'observer' might be him himself. We can interpret the utility of a given image as the psychological benefit/cost of adhering to or deviating from the DM's ideal preferences. For example, a DM might *want* to be a charitable person but also not want to give up on personal consumption. In situations where his hands are tied—when there is no opportunity to give—he circumvents the psychological cost of selfish behavior. But, when confronted with a choice, he must either forgo direct consumption or address his greed.

While this story can clearly explain choice reversals at second stage choices, it may also make sense in the context of first stage choice. A completely rational and forward looking DM would understand that choosing to avoid the future option of donating money is effectively choosing not to donate, and would therefore be unable to skip out on the phycological bill. Of course, like all benchmarks of rationality, there is a growing body of evidence suggesting humans do not meet this standard: for example see Gino et al. (2016) and Grossman and Van Der Weele (2017). Within the present context, the interpretation being that a phycological cost is levied only when consumption decisions are actually made, so DM might avoid situations where donating to a charity is possible, even if donating nothing is always an option.

A Axiomatization of Axiom A1

We need a value function:

Axiom 3—WARP. If $D, D' \in \mathcal{M} \cap \mathcal{M}'$, $D \in \mathscr{C}_1(\mathcal{M})$ and $D' \in \mathscr{C}_1(\mathcal{M}')$ then $D \in \mathscr{C}_1(\mathcal{M}')$.

Even if we do not insist that the preference over images is continuous, the fact that the DM's preference over consumption objects is continuous requires that when considering only degenerate 2CPs, the DM has a continuous preferences—the projection of contour sets onto $\mathcal{D}^1$ must be closed.

Axiom 4—Weak Continuity. For all $D \in \mathcal{D}$, $UC(D) \cap \mathcal{D}^1$ and $LC(D) \cap \mathcal{D}^1$ are closed and non-empty.

The non-emptiness restriction ensures that images are not lexicographically preferred to dis-preferred to one another. For example, if $UC(D) \cap \mathcal{D}^1$ was empty, then there no consumption object, no matter how good, that is preferred (along with the trivial image) to D. Because we are interested in linear utilities over consumption objects, this would indicate that the image associated with the choice from D is infinitely good.[4]

When all image concerns are obviated, we want preferences to be expected utility.

Axiom 5—Singleton Independence. For all $\lambda \in \mathbb{R}_{++}$, $\mathcal{M}, \mathcal{M}' \subset \mathcal{D}^1$,

$$\mathscr{C}_1(\lambda\mathcal{M} + \lambda'\mathcal{M}') = \lambda\mathscr{C}_1(\mathcal{M}) + \lambda'\mathscr{C}_1(\mathcal{M}').$$

These axioms provide the scaffolding of the representation, a value function over choice problems that, when looking at degenerate problems, reflects a linear preference over consumption objects.

Lemma 1. *If $\langle \mathscr{C}_1, \mathcal{C}_2 \rangle$ satisfies A3—5 if and only if it satisfies A1. (For the readers connivence: A1 imposes the existance a translation invariant value function, $V : \mathcal{D} \to \mathbb{R}$ representing $\mathscr{C}_1$ such that $u : x \mapsto V(\{x\})$ is a linear function over $\mathbb{R}^n$.)*

Proof. Necessity is standard. Towards sufficiency, consider the projection of $\mathscr{C}_1$ to $\mathcal{M} \subset \mathcal{D}^1$. Over this space, $\mathscr{C}_1$ satisfies the expected utility axioms. Therefore, there exists a linear $u : \mathbb{R}^n \to \mathbb{R}$ rationalizing the projection of $\mathscr{C}_1$.

Now consider any $D \in \mathcal{D}$. We claim that $UC(D) \cap LC(D) \cap \mathcal{D}^1$ is non-empty—the Lemma then follows directly by setting $V(D) = u(x)$ for an $\{x\}$ in the intersection. Assume

[4] If we want, in addition, the value function over images to be continuous, we can strengthen A4 as follows:

Axiom 4*—k-Weak Continuity. For all $D \in \mathcal{D}$ and $k \in \mathbb{N}$, $UC(D) \cap \mathcal{D}^k$ and $LC(D) \cap \mathcal{D}^k$ are closed.

Notice that even when the DM's utility over images is continuous, $\mathscr{C}_1$ is *not* continuous (in the relevant topology) because the map carrying a choice to its associated image is not itself continuous. To see this, notice that if $D_\lambda = \{x, y\}$ for $x \neq y$. For $\lambda \in (0,1)$, $I^{\lambda x + \lambda' x}_{\lambda D + \lambda' x} \neq \mathbb{R}^n$ whereas the limiting choice indices the image $I^x_{\{x\}} = \mathbb{R}^n$. Such complications arise whenever two elements collide in the limit, a problem which does not happen when restricting the domain to $\mathscr{M}^k$.

the claim did not hold. Take $x \in \arg\max_{LC(D)\cap\mathcal{D}^1} u(x)$ and $y \in \arg\min_{UC(D)\cap\mathcal{D}^1} u(x)$ which exist and are distinct by A4 and our assumption. By A3 and the linearity of u, $u(x) < u(\frac{1}{2}x + \frac{1}{2}y) < u(y)$. Thus, $\{\frac{1}{2}x + \frac{1}{2}y\}$ is in neither the upper nor the lower contour set of D, a contradiction to the non-emptiness of $\mathscr{C}_1$. ∎

B Single Stage Image Conscious Choice

The interpretation of two stage choice is that $\mathscr{C}_1$ represents a choice over 2CPs that is made in the absence of image concerns. Hence, in many scenarios, this choice function will not be observable. This section considers the image conscious model when only second stage choice is accessible to the modeler; it posits axioms only on $\mathcal{C}_2$ equivalent to (**??**) of the IC representation.

Limited observability bears a cost. First, the effective uniqueness of Γ is no longer ensured. Second, the axiomatic structure and concomitant proof rely more directly on technical assumptions, and so, are correspondingly more involved. This latter point is self evident, but to understand the failure of uniqueness, consider the following.

Say $I, J \in \mathbb{I}$ are *directly* comparable if there is a D such that $I = I^x_D$, $J = I^y_D$ and either x or y is in $\mathcal{C}_2(D)$. When I and J are directly comparable (and, say x is chosen), we have an bound on the utility difference between I and J in terms of consumption utility:

$$\Gamma(I) - \Gamma(J) \geq u(y) - u(x).$$

Say that $I, J \in \mathbb{I}$ are *indirectly comparable* if they are contained in the transitive closure of the direct comparability relation. If two images are not indirectly comparable, then there is no restriction imposed by the observed choices on the relative values of the images. Indirect comparability is an equivalence relation; Γ in the resulting representation can be normalized independently across the classes of this equivalence relation.

B.1 Axioms

Scaling a choice problem may result in non-linear tradeoffs. As λ increases, choice from λD places more importance on consumption utility. The first axiom allows $\mathcal{C}_2(\lambda D)$ to vary non-linearly in λ, but ensures that deviations are consistent with increasing importance on consumption utility.

Axiom 1°—Scale Acyclicity. Let $0 < \lambda < \lambda' < \lambda''$ and $D \in \mathcal{D}$. If $x \in \frac{1}{\lambda}\mathcal{C}_2(\lambda D) \cap \frac{1}{\lambda''}\mathcal{C}_2(\lambda'' D)$ then $x \in \frac{1}{\lambda'}\mathcal{C}_2(\lambda' D)$.

In the limit, as $\lambda \to \infty$, only consumption utility matters. Indeed, this is the manner in which u might be identified. To get at this, we can define the following map, which is well defined given A1° and the finiteness of each D:

$$\mathcal{C}_2^\infty : D \mapsto \lim_{\lambda\to\infty} \frac{1}{\lambda}\mathcal{C}_2(\lambda D).$$

To identify u we need $\mathcal{C}_2^\infty$ to be well behaved; ideally this would just entail the imposition of WARP. Unfortunately, it is possible that $u(x) = u(y)$ but $y \neq \mathcal{C}_2^\infty(\{x, y\})$; this happens whenever $\Gamma(I^x_{\{x,y\}}) > \Gamma(I^y_{\{x,y\}})$. To deal with this, we impose WARP on perturbed choice problems.

Axiom 2°—Sequential Limit Consistency. Let $\{D_k\}_{k\in\mathbb{N}} \subset \mathcal{D}$ converge to $D \supseteq \{y, x\}$ such that $x \in \mathcal{C}_2^\infty(D_k)$ for all $k \in \mathbb{N}$. Then for any D' with $y \in \mathcal{C}_2^\infty(D')$ there exists a sequence $D'_k \to D'$ such that $x \in \mathcal{C}_2^\infty(D'_k \cup \{x\})$ for all k.

Translation invariance (A2) remains, but is transcribed here for completeness.

Axiom 3°—Translation Invariance. For all $x \in \mathbb{R}^n$ and $D \in \mathcal{D}$,

$$\mathcal{C}_2(D + x) = \mathcal{C}_2(D) + x$$

With these three axioms, u can be identified.

Lemma 2. *If $\mathcal{C}_2$ satisfies A1°—3° then there exists a linear $u : \mathbb{R}^n \to \mathbb{R}$ such that*

$$\mathcal{C}_2^\infty(D) \subseteq \arg\max_D u.$$

Moreover, u is unique up to positive linear transformations.

Proof. In section B.2. ∎

From $\mathcal{C}_2$ we can define $\succcurlyeq \subset (\mathbb{R}^n \times \mathbb{I}) \times (\mathbb{R}^n \times \mathbb{I})$ via $(x, I) \succcurlyeq (y, J)$ iff there exists a $D \supseteq \{x, y\}$ with $I_D^x = I$ and $I_D^y = J$, and such that $x \in \mathcal{C}_2(D)$. The next axioms place restrictions on $\succcurlyeq$ but these can be translated back into choice behavior in the obvious, but tedious, manner. Per normal let $\succ$ and $\sim$ denote the asymmetric and symmetric components.

The relation $\succcurlyeq$ will necessarily be highly incomplete; for example, images with overlapping relative interiors will never be comparable. Because of this, $\succcurlyeq$ will not be transitive; it should, however, be extendable to a transitive relation.

Axiom 4°—Acyclicity. $\succ$ is acyclic.

Finally, we impose three restrictions that relate the choice over $\succcurlyeq$ to the consumption utility as identified by Lemma 2: *monotonicity* states that if $(x, I) \succcurlyeq (y, J)$ and $u(x') > u(x)$ then not $(y, J) \succ (x', I)$—ceteris paribus, more consumption is better; *boundedness* states that $(x, I) \succ (y, J)$ cannot hold for all x—I cannot be 'infinitely' better than J; *continuity* states that if $u(x_n) \to u(x)$ and $(x_n, I) \succcurlyeq (y, J)$ for all n, then not $(y, J) \succ (x, I)$—preferences cannot be reversed in the limit.

In the proof of the representation theorem, we will extend $\succcurlyeq$ to a complete binary relation, showing these properties still hold; as such, it is helpful to define things for a general relation R defined over $(\mathbb{R}^n \times \mathbb{I})$.

Definition. Let $v : \mathbb{R}^n \to \mathbb{R}$. Call a relation R (with asymmetric component S) defined over $(\mathbb{R}^n \times \mathbb{I})$ *v-monotone* if whenever

1. $v(z) > 0$ and $(x, I)R(y, J)$, or,
2. $v(z) \geq 0$ and $(x, I)S(y, J)$,

then not $(y, J)R(x + z, I)$.

Definition. Let $v : \mathbb{R}^n \to \mathbb{R}$. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *v-bounded* if for all $I, J \in \mathbb{I}$, it is true that $\inf\{v(z) \mid (z, I)R(\mathbf{0}, J)\} > -\infty$.

Definition. Let $v : \mathbb{R}^n \to \mathbb{R}$. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *v-continuous* if for all $I, J \in \mathbb{I}$, if whenever there exists a sequence $\{x_k\}_{k \in \mathbb{N}}$ such that $v(x_n) \to 0$ and $(x_k, I)R(y, J)$ for all k, then for any x with $v(x) = 0$, if $(y, J)R(x, I)$ then $(x, I)R(y, J)$.

Let $\succcurlyeq^{TC}$ denote the transitive closure of $\succcurlyeq$.

Axiom 5°—Consumption Regularity. The relation $\succcurlyeq^{TC}$ is u-monotone, u-continuous, and u-bounded.

These axioms are equivalent to the existence of a image conscious representation $\langle u, \Gamma\rangle$ which represents $\mathcal{C}_2$ as (**??**).

Theorem B.1. *The following are equivalent:*

1. *$\mathcal{C}_2$ satisfies* A1°—5°
2. *$\mathcal{C}_2$ has an image conscious representation $\langle u, \Gamma\rangle$.*

Moreover, u is unique up to positive linear translations and Γ is unique up-to an additive constant within each equivalence class generated by the indirect comparability relation.

Proof. In section B.3. ∎

In contrast to the proof of Theorem 3.1, the proof of Theorem B.1 is rather involved. The main difficulty surrounds the intrinsic incompleteness of the induced preference relation on $\mathbb{R}^n \times \mathbb{I}$, owing to the geometric dependence between the set of consumption alternatives and the consequent images. Indeed, imagine that some complete $\succcurlyeq^\star$ over $\mathbb{R}^n \times \mathbb{I}$ was magically identified and preserved the relevant structure and extended $\succcurlyeq$. Then, fixing $I^* \in \mathbb{I}$ and setting $\Gamma(I^*) = 0$, we can recover the entirety of Γ is by simply setting:

$$\Gamma : I \mapsto -u(x^I)$$

where x^I is a consumption alternative such that $(x^I, I) \sim (\mathbf{0}, I^*)$. Such an alternative exists by the u-boundedness and u-continuity assumptions, and its utility is unique by u-monotonicity. Translation invariance and transitivity then ensure the resulting $\langle u, \Gamma\rangle$ actually represents $\succcurlyeq^\star$, and hence $\mathcal{C}_2$.

Guaranteeing that $\succcurlyeq$ can be extended to a complete $\succcurlyeq^\star$ (while preserving the axiomatic structure) turns out to be pain, but mostly for technical reasons. The relatively simple core idea is as follows: we can first extend $\succcurlyeq$ by adding comparisons that were not observed by $\mathcal{C}_2$ but must hold because of transitivity, monotonicity, or continuity. The resulting relation extends $\succcurlyeq$ because of A4° and A5°. Still, there will be images I and J such that no x satisfies $(x, I) \sim (\mathbf{0}, J)$. What can we do? Just pick some x and extend the relation by adding $(x, I) \sim (\mathbf{0}, J)$ (and then again adding all the consequences of transitivity, monotonicity, or continuity). Repeating the process for different I's and J's creates a partial order of extensions of $\succcurlyeq$, which, by Zorn's Lemma, has maximal element that must be complete.[5]

This process also elucidates the exact nature of non-uniqueness. If two images are initially comparable, that is there exists an x and y such that $(x, I) \sim (y, J)$ is implied by the initial choice function, then the difference between $\Gamma(I)$ and $\Gamma(J)$ is identified (up to a common normalization) by the difference between $u(x)$ and $u(y)$. Thus, identification is made over the equivalence classes of initially comparable images (that comparability is an equivalence relation is Lemma 5(i)), but, these equivalence classes can be independently normalized.

B.2 Proof of Lemma 2

Define the preference relation, $\dot{\succcurlyeq}$, on $\mathbb{R}^n$ as follows: $x \dot{\succcurlyeq} y$ if there exists a $\{D_k\}_{k\in\mathbb{N}} \subset \mathcal{D}$ converging to $D \supseteq \{y, x\}$ such that $x \in \mathcal{C}_2^\infty(D_k)$ for all k. We claim that $\dot{\succcurlyeq}$ is an expected utility preference; this would complete the lemma, for if $x \in \mathcal{C}_2^\infty(D)$ then $x \dot{\succcurlyeq} y$ by taking the constant sequence D, and hence $x \in \arg\max_D u$ for any representation of $\dot{\succcurlyeq}$.

[5] This is one of the many instantiations of Szpilrajn's extension theorem with additional structure being preserved by the extension.

Completeness. Fix $x, y \in \mathbb{R}^n$, and take a sequence $\{y_k\}_{k\in\mathbb{N}}$ converging to y. Since $\mathcal{C}_2$ is non-empty there exists a subsequence (w.l.o.g., indexed by the same k) such that for all k either $x \in \mathcal{C}_2^\infty(\{x, y_k\})$ or $x \notin \mathcal{C}_2^\infty(\{x, y_k\})$. If it is the former, we are done and $x \dot{\succcurlyeq} y$. If it is the latter, we can appeal to translation invariance, and for each k, shift by $y - y_k$ to obtain a sequence $\{x + y - y_k, y\}$ such that y is always chosen, so $y \dot{\succcurlyeq} x$.

Transitivity. Let $x \dot{\succcurlyeq} y$ and $y \dot{\succcurlyeq} z$. Consider the choice problem $D = \{x, y, z\}$. If $x \in \mathcal{C}_2^\infty(D)$ then $x \dot{\succcurlyeq} z$ and we are done. If $y \in \mathcal{C}_2^\infty(D)$, then we can appeal to A2° to obtain a sequence $D_k \to D$ such that $x \in \mathcal{C}_2^\infty(D_k \cup \{x\})$ for all k, hence $x \dot{\succcurlyeq} z$ (notice, $x \dot{\succcurlyeq} y$ definitionally implies the antecedent for A2°). Finally, assume $z \in \mathcal{C}_2^\infty(D)$. Then by the above reasoning, we have a sequence $D_k \to D$ such that $y \in \mathcal{C}_2^\infty(D_k \cup \{y\})$. Now since $x \dot{\succcurlyeq} y$, we can, for each D_k find a further sequence $D^k_{k'} \to D_k \cup \{y\}$ such that $x \in \mathcal{C}_2^\infty(D^k_{k'} \cup \{x\})$ for all $k, k' \in \mathbb{N}$. For each $m \in \mathbb{N}$, set $\hat{D}_m$ to be the first element of $\{D^m_{k'}\}_{k'\in\mathbb{N}}$ such that $d_H(D^m_{k'} - D_m) \leq \frac{1}{m}$. This is a sequence converging to D and with x always chosen.

Continuity. Let $\{y_k\}_{k\in\mathbb{N}}$ converge to y and be such that $x \dot{\succcurlyeq} y_k$ for all k. Then by definition, we have a sequence of sequences $\{\{D^k_{k'}\}_{k'\in\mathbb{N}}\}_{k\in\mathbb{N}}$ such that $D^k_{k'} \to D_k$ for all k and $x \in \mathcal{C}_2^\infty(D^k_{k'} \cup \{x\})$. As above, we can find a sequence of sets converging to $\{x, y\}$ such that x is chosen from each. Closure of the lower contour sets is the analogous.

Independence. Let $x \dot{\succcurlyeq} y$; we have $\{D_k\}_{k\in\mathbb{N}} \subset \mathcal{D}$ converging to $D \supseteq \{y, x\}$ such that $x \in \mathcal{C}_2^\infty(D_k)$ for all k. Set $\lambda \in (0, 1)$ and $z \in \mathbb{R}^n$. We know $x \in \mathcal{C}_2^\infty(D_k)$ indicates by definition that $x \in \lim\limits_{\gamma\to\infty} \frac{1}{\gamma}\mathcal{C}_2(\gamma D_k) = \frac{1}{\lambda} \lim\limits_{\gamma\to\infty} \frac{1}{\gamma}\mathcal{C}_2(\gamma\lambda D_k)$ or, multiplying by λ, that $\lambda x \in \mathcal{C}_2^\infty(\lambda D_k)$. Then by A2 we have that $\lambda x + \lambda' z \in \mathcal{C}_2^\infty(\lambda D_k + \lambda' z)$. Since $\lambda D_k + \lambda' z$ converges to $\lambda D + \lambda' z$, we have that $\lambda x + \lambda' z \dot{\succcurlyeq} \lambda y + \lambda' z$, as desired. ∎

B.3 Proof of Theorem B.1

Definition. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *translation invariant* if for all $x, y, z \in \mathbb{R}^n$ and $I, J \in \mathbb{I}$ we have $(x, I)R(y, J)$ if and only if $(x + z, I)R(y + z, J)$.

Since $\mathcal{C}_2$ is translation invariant, $\succcurlyeq^{TC}$ is as well.

Definition. Let R_1 and R_2 denote two binary relations on a set X (with asymmetric components S_1 and S_2). We say that R_1 *extends* R_2 if $R_2 \subseteq R_1$ and if xS_2y then also xS_1y.

That is, R_1 includes all comparisons that R_2 includes, but does not break any asymmetric comparison into a symmetric one. Because $\succ$ is acyclic, $\succcurlyeq^{TC}$ extends $\succcurlyeq$.

Definition. Let $v : \mathbb{R}^n \to R$. Call a relation R (with asymmetric component S) defined over $(\mathbb{R}^n \times \mathbb{I})$ *strongly-v-monotone* if $(x+z, I)R(x, I)$ whenever $v(z) \geq 0$ and $(x+z, I)S(x, I)$ whenever $v(z) > 0$.

Notice that a transitive and strongly-v-monotone relation is also v-monotone. Let $\succcurlyeq^{\#}$ denote $\succcurlyeq^{TC} \cup \{((x + z, I), (x, I)) \mid x, z \in \mathbb{R}^n, u(z) \geq 0, I \in \mathbb{I}\}$ and $\succcurlyeq^*$ its transitive closure.

Lemma 3. *$\succcurlyeq^*$ is reflexive, transitive, translation invariant, strongly-u-monotone, u-bounded, u-continuous and extends $\succcurlyeq$.*

Proof. That $\succcurlyeq^*$ is reflexive follows from the addition of $((x+\mathbf{0}, I), (x, I))$; that it is transitive is immediate in that it is a transitive closure; that it is translation invariant follows from that translation invariance of $\succcurlyeq^{TC}$ and the fact that all added relations are added in a translation invariant way. Next, notice that $\succcurlyeq^{\#}$ is obviously u-monotone and u-bounded.

Further, notice that, because of u-monotonicity, the addition comparisons added to $\succsim^{TC}$ cannot turn a strict preference into an indifference; hence $\succsim^{\#}$ extends $\succsim^{TC}$.

$\succsim^*$ EXTENDS $\succsim^{TC}$. Assume this was not the case so that we have a finite sequence

$$(x_1, I_1) \succsim^{\#} (x_2, I_2) \succsim^{\#} \ldots \succsim^{\#} (x_m, I_m)$$

such that $(x_m, I_m) \succ^{TC} (x_1, I_i)$.

Notice that for at least one $j < m$ we have

$$(x_{j+1}, I_{j+1}) = (x_j - z'_j, I_j) \tag{B.1}$$

for some z'_j with $u(z'_j) \geq 0$. If this was not the case, then each relation holds also for $\succsim^{TC}$, indicating that $(x_1, I_1) \succsim^{TC} (x_m, I_m)$, a clear contradiction.

So, let $B \subseteq \{1 \ldots m\}$ denote the non-empty set of indices where (B.1) holds for some $z'_j \in \mathbb{R}^n$ with $u(z'_j) \geq 0$. We have:

$$(x_1, I_1) \succsim^{\#} \ldots \succsim^{\#} (x_j, I_j) \succsim^{\#} (x_{j+1}, I_{j+1}) = (x_j - z'_j, I_j) \succsim^{\#} \ldots \succsim^{\#} (x_m, I_m)$$

By translation invariance, we can, for the lowest $j \in B$, add z'_j from the all terms after $j+1$ to obtain

$$(x_1, I_1) \succsim^{\#} \ldots \succsim^{\#} (x_{j-1}, I_{j-1}) \succsim^{\#} (x_j, I_j) = (x_{j+1} + z'_j, I_j) \succsim^{\#} \ldots \succsim^{\#} (x_m + z'_j, I_m)$$

Continuing to delete terms in this manner for all $i \in B$, we are left with a sequence, contained within $\succsim^{TC}$, asserting $(x_1, I_1) \succsim^{TC} (x_m + \sum_{i \in B} z'_i, I_m)$, contradicting u-monotonicity.

STRONG-u-MONOTONICITY. By way of contradiction, assume that by taking the transitive closure we generate a violation of strong-u-monotonicity. That $(x + z, I) \succsim^* (x, I)$ is immediate, so assume this holds only weakly: for some (x, I), $(x, I) \succsim^* (x + z, I)$ for $z \in \mathbb{R}^n$ with $u(z) > 0$.

This requires a sequence of comparisons

$$(x, I) = (x_1, I_1) \succsim^{\#} (x_2, I_2) \succsim^{\#} \ldots \succsim^{\#} (x_m, I_m) = (x + z, I)$$

As above, let $B \subseteq \{1 \ldots m\}$ denote the set of indices where (B.1) holds for some $z'_j \in \mathbb{R}^n$ with $u(z'_j) > 0$. If $B = \emptyset$, we could exhibit this sequence via $\succsim^{TC}$, violating u-monotonicity. Therefore, as above, we can appeal to translation invariance to delete terms for each $i \in B$: the resulting sequence is contained within $\succsim^{TC}$ and asserts $(x, I) \succsim^{TC} (x + z + \sum_{j \in B} z'_j, I)$, contradicting u-monotonicity.

u-BOUNDEDNESS. Fix $z \in \mathbb{R}^n$ and $I, J \in \mathbb{I}$. Let $(z, I) \succsim^* (\mathbf{0}, J)$ so that there exists a finite sequence

$$(z, I) = (x_1, I_1) \succsim^{\#} (x_2, I_2) \succsim^{\#} \ldots \succsim^{\#} (x_m, I_m) = (\mathbf{0}, J)$$

Once again, let $B \subseteq \{1 \ldots m\}$ denote the set of indices where (B.1) holds for some $z'_j \in \mathbb{R}^n$ with $u(z'_j) > 0$. If $B = \emptyset$, then this sequence would exist within $\succsim^{TC}$, indicating $\inf\{u(y) \mid (y, I) \succsim^{TC} (\mathbf{0}, J)\} \leq u(z)$. If B is not empty, we can proceed by the usual trick to conclude $(z, I) \succsim^{TC} (\sum_{j \in B} z'_j, J)$, or by translation invariance, $(z - \sum_{j \in B} z'_j, I) \succsim^{TC} (\mathbf{0}, J)$. This indicates that $\inf\{u(y) \mid (y, I) \succsim^{TC} (\mathbf{0}, J)\} \leq u(z) - \sum_{j \in B} u(z'_j) \leq u(z)$. Since $u(z)$ was arbitrary, the infimum with respect to $\succsim^*$ can be no lower than with respect to $\succsim^{TC}$, which was bounded below.

u-CONTINUITY. Let x be such that $u(x) = 0$ and a sequence $\{x_k\}_{k \in \mathbb{N}}$ converging with $u(x_n) \to 0$. Let (y, J) be such that $(x_k, I) \succsim^* (y, J)$ for all k and $(y, J) \succsim^* (x, I)$. We can use the now standard trick to find following relations:

$$(x_k - z_k, I) \succsim^{TC} (y, J)$$

for each k, and

$$(y, J) \succcurlyeq^{TC} (x + z, I)$$

with $u(z_k) \geq 0$ for each k and $u(z) \geq 0$. Necessarily, $u(z) = 0$, or else, eventually $u(x_k - z_k) < u(x + z)$ creating a violation of u-monotonicity. For the same reason, it must be that for all $u(z_k) \leq u(x_k)$. Hence $u(x_k - z_k) \to 0$, and by u-continuity $(x + z, I) \succcurlyeq^{TC} (y, J)$. Now, since $u(-z) = 0$ we have that $(x, I) \succcurlyeq^{\#} (x + z, I) \succcurlyeq^{\#} (y, J)$, and hence, $(x, I) \succcurlyeq^{*} (y, J)$. ★

Definition. Let $v : \mathbb{R}^n \to R$. Call a relation R defined over $(\mathbb{R}^n \times \mathbb{I})$ *strongly-v-continuous* if for all $I, J \in \mathbb{I}$, if whenever there exists a sequence $\{x_k\}_{k \in \mathbb{N}}$ such that $v(x_n) \to 0$ and $(x_k, I) R (y, J)$ for all k, then for any x with $v(x) = 0$, $(x, I) R (y, J)$.

Lemma 4. *Let $\succcurlyeq^{+}$ be the transitive closure of*

$$\succcurlyeq^{*} \cup \Big\{ ((x, I), (y, J)) \mid \textit{Exists } \{z_k\}_{k \in \mathbb{N}}, u(z_n) \to 0, (x + z_k, I) \succcurlyeq^{*} (y, J) \textit{ for all } k \Big\}$$

Then $\succcurlyeq^{+}$ is reflexive, transitive, translation invariant, strongly-u-monotone, u-bounded, strongly-u-continuous and extends $\succcurlyeq^{*}$ (hence $\succcurlyeq$).

Proof. Reflexivity, transitivity, translation invariance, and strong-u-continuity are all immediate.

$\succcurlyeq^{+}$ EXTENDS $\succcurlyeq^{*}$. Let $(y, J) \succcurlyeq^{+} (x, I)$. Then there must exist a sequence $\{(x_j, I_j)\}_{j=1}^{m}$, with $(x_1, I_1) = (y, J)$ and $(x_m, x_m) = (x, I)$, and such that for each $j < m$ there is a sequence $\{z_k^j\}_{k \in \mathbb{N}}, u(z_k^j) \to 0$ (possibly the constant sequence $\mathbf{0}$, if $(x_i, I_i) \succcurlyeq^{*} (x_{i+1}, I_{i+1})$) such that $(x_j + z_k^j, I_j) \succcurlyeq^{*} (x_{j+1}, I_j)$ for all k. It is without loss of generality to assume that $u(z_k^j) \geq 0$ for all j, k. But notice we have

$$(x_1 + \sum_{i=1}^{m} z_k^i, I_1) \succcurlyeq^{*} (x_2 + \sum_{i=2}^{m} z_k^i, I_2) \succcurlyeq^{*} \ldots (x_j + \sum_{i=j}^{m} z_k^i, I_j) \succcurlyeq^{*} \ldots \succcurlyeq^{*} (x_m, I_m)$$

for each k. This indicates that $(y + \sum_{i=1}^{m} z_k^i, J) \succcurlyeq^{*} (x, I)$ where $u(\sum_{i=1}^{m} z_k^i) \to 0$. So by the u-continuity of $\succcurlyeq^{*}$, we cannot have $(x, I) \succ^{*} (y, J)$: therefore $\succcurlyeq^{+}$ extends $\succcurlyeq^{*}$.

STRONG-u-MONOTONICITY. We have that $(x, I) \succcurlyeq^{+} (x + z, I)$ immediately; since $\succcurlyeq^{+}$ extends $\succcurlyeq^{*}$ it cannot be that $(x + z, I) \succcurlyeq^{+} (x, I)$.

u-BOUNDEDNESS. Fix $x \in \mathbb{R}^n$ and $I, J \in \mathbb{I}$. Let $(x, I) \succcurlyeq^{+} (\mathbf{0}, J)$. Using the same trick as in the proof of extension, we can find a (finite) collection of sequences, $\{\{z_k^j\}_{k \in \mathbb{N}}\}_{j=1}^{m}$ such that $u(z_k^j) \to 0$ for each j and $(x + \sum_{i=1}^{m} z_k^i, I) \succcurlyeq^{*} (\mathbf{0}, J)$. Since $u(x + \sum_{i=1}^{m} z_k^i) \to u(x)$ we have that $u(x) \geq \inf\{v(z) \mid (z, I) \succcurlyeq^{*} (\mathbf{0}, J)\}$. ★

Lemma 5. *Let $v : \mathbb{R}^n \to \mathbb{R}$ be a linear function and R be a preorder on $(\mathbb{R}^n \times \mathbb{I})$ that is translation invariant, strongly-v-monotone, v-bounded and strongly-v-continuous. Call $I, J \in \mathbb{I}$* R-comparable *if there exists an $x \in \mathbb{R}^n$ such that $(x, I) R (\mathbf{0}, J)$ and $(\mathbf{0}, J) R (x, I)$. Then*

1. *R-comparability is an equivalence relation.*

2. *If I, J are not comparable, then there exists $\bar{x} \in \mathbb{R}^n$ such that neither $(\bar{x}, I) R (\mathbf{0}, J)$ nor $(\mathbf{0}, J) R (\bar{x}, I)$*

3. *If $\bar{I}, \bar{J}$ are not comparable, and $\bar{x}$ is as in (2), then, R^* defined as the transitive closure of $R^{\#} = R \cup \{(\bar{x} + z, \bar{I}) R (z, \bar{J}), (z, \bar{J}) R (\bar{x} + z, \bar{I}) \mid z \in \mathbb{R}^n\}$ is also a translation invariant, strongly-v-monotone, v-bounded, and strongly-v-continuous preorder that extends R.*

Proof. (1) Reflexivity is immediate. Symmetry follows from translation invariance. Transitivity follows from the transitivity and translation invariance of R, in the obvious way.

(2) Consider the sets $\{v(x) \mid (x,I)R(\mathbf{0},J)\} \subseteq \mathbb{R}$ and $\{v(x) \mid (\mathbf{0},J)R(x,I)\} \subseteq \mathbb{R}$. By strong-$v$-monotonicity, these are (possibly empty) intervals, the former upward-closed and the later downward-closed. By v-boundedness neither is $\mathbb{R}$ itself. By strong-v-continuity they are closed. If these intervals overlap, then I and J are comparable, so assume they do not overlap. Since $\mathbb{R}$ is connected, so there must be a point not in either interval.

(3) Fix $\bar{I}, \bar{J}$ that are not comparable for some R. Let $R^{\#}$ and R^* be as in the statement of the Lemma, and let and S, $S^{\#}$ and S^* denote respective asymmetric components. Reflexivity, transitivity, and translation invariance are immediate.

R^* EXTENDS R. Assume it did not: there exists a (x,I) and (y,J) such that $(x,I)S(y,J)$ but $(y,J)R^*(x,I)$. This last relations indicates the existence of a sequence,

$$(y,J)R^{\#}(x_1,I_1)R^{\#}\dots R^{\#}(x_m,I_m)R^{\#}(x,I).$$

As in the proof of Lemma 3, there must be some relation not contained in R, so that for some $j < m$, we have $(x_j,I_j) = (\bar{x}+z,\bar{I})$ and $(x_{j+1},I_{j+1}) = (z,\bar{J})$ (or vice versa, with an analogous proof following). It is without loss of generality that there is a single index j such that $(x_j,I_j),(x_{j+1},I_{j+1}) \notin R$.[6] Capitalizing on the fact that R is transitive, we can further delete all other relations, we have

$$(y,J)R(\bar{x}+z,\bar{I})R^{\#}(z,\bar{J})R(x,I).$$

We can split the above sequence and swapping the order, recall $(x,I)S(y,J)$, leaving us with:

$$(z,\bar{J})R(x,I)S(y,J)R(\bar{x}+z,\bar{I}).$$

By the translation invariance of R, this implies $(\mathbf{0},\bar{J})R(\bar{x},\bar{I})$, a contradiction to the definition of $\bar{I},\bar{J}$ and $\bar{x}$.

STRONG-v-MONOTONICITY. We have that $(x,I)R^*(x+z,I)$ immediately; since R^* extends R it cannot be that $(x+z,I)R^*(x,I)$.

v-BOUNDEDNESS. Fix $I,J \in \mathbb{I}$. Define the following constants.

$$a_1 = \inf\{v(z) \mid (z,I)R(\mathbf{0},J)\}$$
$$a_2 = \inf\{v(z) \mid (z,I)R(\bar{x},\bar{I})\}$$
$$a_3 = \inf\{v(z) \mid (z,\bar{J})R(\mathbf{0},J)\}$$

Let $(x,I)R^*(\mathbf{0},J)$. If this relation can be exhibited by R, then $u(x) \le a_1$. So, to make things interesting, assume it cannot be; by the above arguments we can find the following sequence of relations:

$$(x,I)R(\bar{x}+z',\bar{I})R^{\#}(z',\bar{J})R(\mathbf{0},J)$$

By the definition of a_2, and translation invariance, the first relation indicates that $u(x) \ge a_2 + v(z')$. The last relation likewise indicates that $u(z') \ge a_3$; hence $u(x) \ge a_2 + a_3$. In either case, $u(x) \ge \min\{a_1, a_2+a_3\}$ and is hence bounded from below.

[6] To see why: consider the following sequence

$$(\bar{x}+z,\bar{I})R^{\#}(z,\bar{J})R(x_i,I_i)R\dots(x_{i+j},I_{i+j})R(\bar{x}+z',\bar{I})R^{\#}(z',\bar{J})$$

where the "…" contains only R relations. If $v(z') < v(z)$, then $(z,\bar{J})S(z',\bar{J})$ by strong-v-monotonicity, and we can make the same inference deleting one $R^{\#}$ relation. If $v(z') \ge v(z)$ we have a contradiction: we have

$$(z',\bar{J})R(z,\bar{J})R(x_i,I_i)R\dots(x_{i+j},I_{i+j})R(\bar{x}+z',\bar{I})$$

where the first relation is from strong-v-monotonicity. This implies, however, via translation invariance, that $(\mathbf{0},\bar{J})R(\bar{x},\bar{I})$, a contradiction to the definition of $\bar{I},\bar{J}$ and $\bar{x}$.

u-CONTINUITY. Let x be such that $u(x) = 0$ and a sequence $\{x_k\}_{k\in\mathbb{N}}$ converging with $u(x_n) \to 0$. Let (y, J) be such that $(x_k, I)R^*(y, J)$ for all k. By taking a subsequence if necessary, it is without loss of generality to restrict attention to the case where either $(x_k, I)R(y, J)$ for all k or not $(x_k, I)R(y, J)$ for all k. The former is a direct application of the strong-u-continuity of R. Assume the latter: we have,

$$(x_k, I)R(\bar{x} + z, \bar{I})R^{\#}(z, \bar{J})R(y, J)$$

By the strong-u-continuity of R, $(x, I)R(\bar{x} + z, \bar{I})$ and hence $(x, I)R^*(y, J)$. ★

Lemma 6. *There exists a translation invariant, strongly-u-monotone, u-bounded, and strongly-u-continuous preorder on* $(\mathbb{R}^n \times \mathbb{I})$, $\succcurlyeq^\star$, *that extends* $\succcurlyeq^+$ *such that all* $I, J \in \mathbb{I}$ *are* $\succcurlyeq^\star$ *comparable.*

Proof. Consider the set of all translation invariant, strongly-u-monotone, u-bounded, and strongly-u-continuous preorders on $(\mathbb{R}^n \times \mathbb{I})$ that extend $\succcurlyeq^+$. Say that $R \leq R'$ if R' extends R. Clearly, $\leq$ is a partial order, and every chain (totally ordered subset) is bounded by its union. Hence, we can apply Zorn's lemma to conclude the existence of a maximal (with respect to the extension induced order) relation over $(\mathbb{R}^n \times \mathbb{I})$. Call this relation $\succcurlyeq^\star$. By Lemma 5 part (iii), all I, J are $\succcurlyeq^\star$-comparable, or else we could find a further extension, contradicting the maximality of $\succcurlyeq^\star$. ★

For each $I \in \mathbb{I}$, define let x^I denote an element such that $(x^I, I) \sim^\star (\mathbf{0}, \mathbf{0})$. Then define $\Gamma : \mathbb{I} \to \mathbb{R}$ by

$$\Gamma : I \mapsto -u(x^I) \tag{B.2}$$

We now claim that $\langle u, \Gamma\rangle$ forms a IC representation for $\mathcal{C}_2$. Take a menu $D \in \mathcal{D}$. Assume that $x \in \mathscr{C}_1(D)$. Then $(x, I^x_D) \succcurlyeq (y, I^y_D)$ for all $y \in D$. Since $\succcurlyeq^\star$ extends $\succcurlyeq^+$ (Lemma 6), hence $\succcurlyeq$ (Lemma 3), we have $(x, I^x_D) \succcurlyeq^\star (y, I^y_D)$ for all $y \in D$. Therefore, by definition, and translation invariance,

$$(x - x^{I^x_D}, \mathbf{0}) \sim^\star (x, I^x_D) \succcurlyeq^\star (y, I^y_D) \sim^\star (y - x^{I^y_D}, \mathbf{0}).$$

Moreover, by strong-u-monotonicity this indicates that

$$u(x - x^{I^x_D}) \geq u(y - x^{I^y_D}),$$

or, from the definition of Γ and and the linearity of u,

$$u(x) + \Gamma(I^x_D) \geq u(y) + \Gamma(I^y_D),$$

for all $y \in D$. So $\mathcal{C}_2(D) \subseteq \arg\max_{x\in D} \big(u(x) + \Gamma(I^x_D)\big)$.

Now assume that $x \notin \mathcal{C}_2(D)$. Then there exists a $y \in D$ such that $(y, I^y_D) \succ (x, I^x_D)$. Since $\succcurlyeq^\star$ extends $\succcurlyeq$, we have $(y, I^y_D) \succ^\star (x, I^x_D)$. From repetition of the above with strict preference/inequality we conclude that

$$u(y) + \Gamma(I^y_D) > u(x) + \Gamma(I^x_D),$$

So $\arg\max_{x\in D} \big(u(x) + \Gamma(I^x_D)\big) \subseteq \mathcal{C}_2(D)$ and we have established the existence of an image conscious representation. ∎

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