Supplemental Material to

# Failures of Contingent Thinking

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## A Additional Examples

In this section, we explore provide additional examples of how our theory relates to recent experimental findings.

**Framing effects.** Framing effects, wherein different but logically equivalent descriptions of a choice problem yield different choices, constitute a simple and evident violation of rational decision-making. For example, consider a DM valuing bets over the following statements:

P = "the S&P500 goes up tomorrow", or

Q = "the S&P500 goes up tomorrow and the 1 millionth prime number is greater than 15,000,000."

While P and Q are in fact equivalent, as the additional criterion in Q is tautological, it is far from unreasonable for a DM to strictly prefer betting on the former. As humans are not perfect reasoners, we often fail to observe logical equivalence, and thus do not treat logically equivalent statements as such (Stalnaker, 1991; Lipman, 1999). This is easily captured in our model by a $t$ that maps P and Q to.

While the effect above is likely due to the computational complexity of the statements, framing effects abound for myriad other, often emotional, reasons. For example, Tversky and Kahneman (1981) asked subjects consider the efficacy of a drug differently depending on its description.

- 100 patients took the medicine, and 68 patients found it beneficial, or
- 100 patients took the medicine, and 32 patients saw no improvement.

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Again, such differentiation must stem from the failures of of the DM to identify different descriptions of the same event.

**Ambiguity Aversion.** There is an urn with 100 balls, each of which can be white or black, with unknown proportion. A single ball is drawn from the urn, and subjects evaluate bets based on the language constructed from the following statements:

$$\begin{aligned} \textsc{p} &= \text{"The drawn ball is white."} \\ \neg\textsc{p} &= \text{"The drawn ball is not white."} \\ \textsc{p} \vee \neg\textsc{p} &= \text{"The drawn ball is white or it is not white"} \end{aligned}$$

A common pattern of choice is consistent with $\ell(t(\textsc{p})) = \ell(t(\neg\textsc{p})) < \frac{1}{2}\ell(t(\textsc{p} \vee \neg\textsc{p}))$.

This pattern can be explained by an IOU in which there are states where neither $\textsc{p}$ nor its negation are true: for example $W = \{w_1, w_2, w_3\}$ and $t : \textsc{p} \to w_1$, $t : \neg\textsc{p} \to w_2$ and $t : \{\textsc{p}, \neg\textsc{p}\} \to W$.

Here the subject understands that $\textsc{p}$ and $\neg\textsc{p}$ are mutually exclusive, and further understands that together they are always true. However, the subject is not able to cleanly delineate the boundary between $\textsc{p}$ and $\neg\textsc{p}$, and so, entertains states where neither are considered true.

Notice also that this represents a case where a DM's worldview becomes completely rational subsequent to obtaining some information. Conditioning on $\textsc{w}$ she considers only state $w_1$, believing $\textsc{w}$ and $\textsc{w} \vee \neg\textsc{w}$ to be true; conditioning on $\neg\textsc{w}$ she considers only state $w_3$, believing $\neg\textsc{w}$ and $\textsc{w} \vee \neg\textsc{w}$ to be true. Nonetheless, her ex-ante preferences violate **N** and **D**.

**Redundant Evidence.** In a recent working paper, Garfagnini and Walker-Jones (2023) endow subjects with a lottery that has a $x \in \{0, 1, 2, \ldots, 100\}$ percent chance of paying \$20 and a $1 - x$ percent chance of paying nothing. Subjects do not know $x$ but know that each value of $x$ is equally likely. The experimenters elicit from the subjects their *probability-equivalent*, that is, the minimum $y \in [0, 100]$ such that they are willing to trade the unknown (or compound) lottery for a $y$ percent chance of \$20. This equivalent is elicited before and after providing the subjects with various pieces of information about the initial, endowed, lottery of the form

$$\textsc{p}_n = \text{"}x \text{ is greater than } n\text{"}$$

Within this rather simple environment, subjects displayed clear departures from rational behavior. In particular, they often changed their valuation upon receiving redundant evidence. Specifically, they are initially told $\mathrm{P}_n$, reveal a probability-equivalent $y$, and are subsequently told $\mathrm{P}_{n'}$, with $n' \leq n$, and reveal a *new* probability-equivalent $y' \neq y$.

Here, clearly, the subjects' behavior is inconsistent with an understanding that $\mathrm{P}_n$ implies $\mathrm{P}_{n'}$. After conditioning on $\mathrm{P}_n$, the subjects should consider only states where $x \geq n$ so that the additional revelation of $\mathrm{P}_{n'}$ provided no new information. There are two (not necessarily mutually exclusive) explanations: the first is that in the initial state-space, there are inconsistent states, where $\mathrm{P}_n$ is true, but $\mathrm{P}_{n'}$ is false. The second explanation is that subjects do not update by excluding states, but use some other rationally-limited heuristic.

A further result from the authors suggests strongly that the latter is a contributing factor: subjects often revise their probability-equivalents in the opposite direction as expected. Over 60% of subjects with a baseline probability-equivalent $y$ *lowered* their response to $y' < y$ after being informed of $\mathrm{P}_{20}$ (or erred analogously in the opposite direction). While properly disentangling static vs. dynamic rationality would require a more intentional task, this evidence indicates that reducing uncertainty is not necessarily accompanied by a higher standard of rational thinking.

**What you see is all there is.** The following is a slight adaption of Enke (2020): Subjects are presented with bets regarding 6 independent, uniform draws from $X = \{-3, -2, -1, 1, 2, 3\}$. Subjects observe the first draw and subsequently indicate whether they believe the average to be positive or negative. Thereafter, they observe additional signals by interacting with a computerized information source that (transparently) shares all signals that "align" with their first stated belief (e.g., are positive if the first belief is positive) but not all signals that "contradict" said belief. Subjects then must again state their beliefs about the average. Whenever subjects' first signal is positive, their final stated beliefs tend to be upward biased, as if they were ignoring the information contained in *not observing* a given signal (and conversely for initially negative assessments).

Consider $X^+ = \{1, 2, 3\}$ and $X^- = \{-1, -2, -3\}$, and without loss of generality assume that the initial draw is $x \in X^+$. Further, assume that a participant guesses that the average is above positive.

Consider the following statements

$$\textsc{obs}_n = \text{“}n \text{ signals are observed”}$$

$$\textsc{neg}_{5-n} = \text{“}5-n \text{ signals are negative”}$$

Then it seems that the participant's behavior is being driven by a failure to perceive the equivalence “$\textsc{obs}_n$ if and only if $\textsc{neg}_{5-n}$” for each $n \leq 5$.

To see how this failure of contingent thinking can be captured by an IOU, consider the following state-space: $W = \{\pi : X \to \mathbb{N} \mid \sum_X \pi(x) = 5\}$, where each state is identified with a function $\pi$ such that $\pi(x)$ counts the number of draws of $x$. Thus, $t$ maps the statement, for $x \in X$ and $m \leq 5$,

$$\textsc{d}_{x,m} = \text{“there were } m \text{ draws of } x\text{”}$$

to the event $\{\pi \in W \mid \pi(x) = m\}$. Now consider the statement, for $x \in X^+$ and $m \leq 5$,

$$\textsc{o}_{x,m} = \text{“there were } m \text{ observations of } x\text{”}$$

For a perfectly rational subject this should map to the event $\{\pi \in W \mid \pi(x) = m\}$; however, a participant who does not intuit that the missing signals must have been negative will understand this statement as the event $t : \textsc{o}_{x,m} \mapsto \{\pi \in W \mid \pi(x) \geq m\}$.

So long as the participants are otherwise rational, they will understand the statement $\textsc{obs}_n$ as mapping to the union of all different ways they could have observed $n$ signals, and likewise the statement $\textsc{neg}_{5-n}$ as the union of the different ways of having drawn $5-n$ negative draws. That is:

$$t(\textsc{obs}_n) = \bigcup \Big\{ \bigcap_{x \in X^+} t(\textsc{o}_{x,m_x}) \mid \sum_{x \in X^+} m_x = n \Big\} \qquad \text{and}$$
$$t(\textsc{neg}_{5-n}) = \bigcup \Big\{ \bigcap_{x \in X^-} t(\textsc{d}_{x,m_x}) \mid \sum_{x \in X^-} m_x = 5-n \Big\}$$

It is straightforward to see that for the rational subjects, these two sets coincide, whereas for subjects with the flawed $t$ given above, $t(\textsc{neg}_{5-n}) \subsetneq t(\textsc{obs}_n)$.

**Support Theory.** Tversky and Koehler (1994) collect many examples of *unpacking* wherein DMs estimate the probability of an implicit disjunction (e.g., death from a natural cause) to be less than the sum of its mutually exclusive components (e.g., death from heart disease, cancer, or other natural causes). In other words, subjects assessed the collective probability of a set of specific statements as more likely than a statement comprising their disjunction.

This can be captured by a subject who correctly understands that each component statement is contained in the more general case, but fails to properly account for the mutual exclusivity of statements. Consider the following:

$$\begin{aligned}
\textsc{h} &= \text{“The primary cause of death is heart disease.”} \\
\textsc{c} &= \text{“The primary cause of death is cancer.”} \\
\textsc{h} \wedge \textsc{c} &= \text{“The primary cause of death is heart disease and the primary cause of death is cancer”} \\
\textsc{n} &= \text{“The primary cause of death was natural.”}
\end{aligned}$$

and assume that the subject understands that $\textsc{h}\wedge\textsc{c}$ is impossible, so that $t(\textsc{h}\wedge\textsc{c}) = \varnothing$, and that both $\textsc{h}$ and $\textsc{c}$ imply $\textsc{n}$: $t(\textsc{h}) \subseteq t(\textsc{n})$ and $t(\textsc{c}) \subseteq t(\textsc{n})$.

Now, if $t$ distributes over conjunctions, it follows that $t(\textsc{h}) \cap t(\textsc{c}) = \varnothing$, and so it must be that $\ell(t(\textsc{h})) + \ell(t(\textsc{c})) \leq \ell(t(\textsc{n}))$. However, consider $(W, t)$ with $W = \{w_1, w_2, w_3\}$ and $t : \textsc{h} \mapsto \{w_1, w_2\}, \textsc{c} \mapsto \{w_2, w_3\}$, $\textsc{h} \wedge \textsc{c} \mapsto \varnothing$ and $\textsc{n} \mapsto W$.

Here, $t$ is not $\wedge$-distributive, so that although the subject directly recognizes that $\textsc{h} \wedge \textsc{c}$, she still implicitly considers states where neither statement is ruled out.

# B Axiomatic foundation of SIDEU

Here we provide axioms on a preference relation (also denoted $\succcurlyeq$) over syntactic acts. For simplicity, we assume that the image of each syntactic act is finite. We will consider notation for some special acts: For $\Lambda \subseteq \Phi \subset \mathcal{L}$ and $x \in \mathbb{R}_+$, let $x_\Phi^\Lambda$ denote the acts that maps $\Lambda$ to $x$ and $\Phi \setminus \Lambda$ to 0. That is, the act whose domain is $\Phi$, and which pays $x$ if and only if $\Lambda$. Further, identify $x_\Phi$ with the syntactic bet of $x$ on $\Phi$. Let $\textsc{dom}(f) \in \mathcal{L}$ denote the domain of $f$. For two acts $f$ and $g$ such that $\textsc{dom}(f) = \textsc{dom}(g)$, let $\alpha f + \beta g$, for $\alpha, \beta \in \mathbb{R}_+$, denote the point-wise mixture of $f$ and $g$.

In what follows, it will be helpful to consider properties of the *strict* component of preference: Given $\succcurlyeq$, for any pair of acts $f$ and $g$ write $f \mathrel{\dot{\sim}} g$ if $\{h \mid h \succ f\} = \{h \mid h \succ g\}$, and $f \mathrel{\underset{\cdot}{\sim}} g$ if $\{h \mid f \succ h\} = \{h \mid g \succ h\}$. Finally, write $f \approx g$ if $f \mathrel{\dot{\sim}} g$ and $f \mathrel{\underset{\cdot}{\sim}} g$: that is, if $f$ and $g$ satisfy the exactly same $\succ$-relations. Note that $f \approx g$ is not the same as $f \sim g$; this equivalence does however follow if $\succcurlyeq$ is complete and transitive.

The first axiom is the requirement that the strict component of the preference is an interval order.

**A1**—Strict Interval Order. $\succ$ is continuous irreflexive and transitive and has the interval property: if $f \succ g$ then for all $x \in \mathbb{R}_+$ and $\Phi \in \mathcal{K}$ either $f \succ x_\Phi$ or $x_\Phi \succ g$.

The next axiom, *Interpretability*, states that when we restrict the preference to syntactic bets, this restrictions satisfies the axioms from Section **??**.

**A2**—Interpretability. $\succcurlyeq$, restricted to syntactic bets, is an articulate IDEU preference.

Next, we link the preferences over more complex acts to simple bets by assuming the existence of some (upper) certainty equivalent.

**A3**—Bet Equivalence. For all $f$, and nonnull $\Phi$, there exists a $x \in \mathbb{R}_+$ such that $f \mathrel{\dot{\sim}} x_\Phi$.

The next axiom extends the IDEU representation to more complex acts. Even when the DM aggregates in a nonlinear way across states (i.e., the likelihood assessment is a non probability $\ell$), aggregation of *co-monotone* acts will be linear.

**A4**—Co-Monotone Additivity. For $\Phi$, let $f, g : \Phi \to \mathbb{R}$ be co-monotone. Then

- Let $g \mathrel{\underset{\cdot}{\sim}} y_\Phi$. Then $f \succ x_\Phi$ iff $f + g \succ (x+y)_\Phi$.
- Let $y_\Phi \mathrel{\dot{\sim}} g$. Then $x_\Phi \succ f$ iff $(x+y)_\Phi \succ f + g$.

The axioms dictates how a DM's valuation (i.e., the her upper and lower bound valuations as given by $\dot{\sim}$ and $\underset{\cdot}{\sim}$) respond to expanding the set of contingencies on which an act pays a positive payoff.

**A5**—Expansion Consistency. Let $\Phi, \Gamma \subset \Lambda \in \mathcal{K}$ be disjoint and $\Psi \in \mathcal{K}$ be nonnull. Then:

(UPPER) Let $x^{\Gamma}_{\Phi\cup\Gamma} \mathrel{\dot{\sim}} y_{\Psi}$. Then $z_{\Psi} \succ x^{\Phi}_{\Lambda}$ iff $(z+y)_{\Psi} \succ x^{\Phi\cup\Gamma}_{\Lambda}$.

(LOWER) Let $x^{\Gamma}_{\Lambda\setminus\Phi} \mathrel{\dot{\sim}} y_{\Psi}$. Then $x^{\Phi}_{\Lambda} \succ z_{\Psi}$ iff $x^{\Phi\cup\Gamma}_{\Lambda} \succ (z+y)_{\Psi}$.

When moving from betting on $\Phi$ to betting on $\Phi \cup \Gamma$, the DM's upper-bound valuation increases directly by the value of a bet on $\Gamma$. However, her lower-bound valuation can increase by more, since by changing the payoff contingent on $\Gamma$ from 0 to 1, the act also becomes less ambiguous—if the DM perceived that $\Phi$ and $\Gamma$ might overlap, then the bet on $\Phi$ alone is not well defined in the DM's mind, but additionally betting on $\Gamma$ can alleviate this concern and thereby increase the value of the act by more than the direct value of a bet on $\Gamma$.

THEOREM 1. *A relation $\succcurlyeq$ satisfies* **A1**–**A5** *if and only if it is a sparse IDEU preference.*

*Proof.* We consider only the nontrivial case wherein $\mathcal{L}$ is nonnull; retaining our abuse of notation, associate each $x \in \mathbb{R}_+$ with the act $x_{\mathcal{L}}$. For the remainder of this section, let $(W, t, \ell)$ denote the representation of $\succcurlyeq$ required by **A2**.

LEMMA 2. *Let $\succcurlyeq$ satisfy* **A1** *and* **A2**. *Let $\Phi$ be nonnull and let $f \mathrel{\dot{\sim}} x_{\Phi}$ and $y_{\Phi} \mathrel{\dot{\sim}} g$. Then, $x > y$ if and only if $f \succ g$.*

*Proof.* Set $z = \frac{1}{2}x + \frac{1}{2}y$. If $x > y$, then, since $x_{\Phi} \succ z_{\Phi}$ also $f \succ z_{\Phi}$. Likewise, $z_{\Phi} \succ g$, so by transitivity, $f \succ g$. If $y \geq x$, then $z_{\Phi} \not\succ g$ and $f \not\succ z_{\Phi}$. Therefore, by the contrapositive of the interval property, $f \not\succ g$. ■

LEMMA 3. *Let $\succcurlyeq$ satisfy* **A1**, **A2**, **A3** *and* **A4**. *Then, for $\Phi \in \mathcal{K}$ $f \succ x_{\Phi}$ if and only if $\alpha f \succ \alpha x_{\Phi}$ for $\alpha > 0$ and, likewise: $x_{\Phi} \succ f$ if and only if $\alpha x_{\Phi} \succ \alpha f$ for $\alpha > 0$.*

*Proof.* Let $\frac{1}{2}f \mathrel{\dot{\sim}} y$ and $f \mathrel{\dot{\sim}} x$. These are guaranteed to exist by **A3**. By **A4**, we have $z \succ f$ iff $z - y \succ \frac{1}{2}f$. Hence $y = \frac{1}{2}x$. Repeating as necessary, we have $\frac{1}{2^n}f \mathrel{\dot{\sim}} \frac{1}{2^n}x$. Now, the set of finite sums of (possibly repeating) elements of the set $\{\frac{1}{2^n}\}_{n\in\mathbb{N}}$ is dense in $\mathbb{R}_+$, so additivity and continuity seal the deal. ■

LEMMA 4. *Set some $\Phi \subseteq \Lambda \in \mathcal{K}$. If $\succcurlyeq$ satisfies* **A2** *and* **A5**(LOWER) *then*

$$1^{\Phi}_{\Lambda} \mathrel{\dot{\sim}} \ell(t(\Lambda)) - \ell(t(\Lambda \setminus \Phi)). \tag{1}$$

*If in addition,* $\succcurlyeq$ *satisfies* **A5**(UPPER) *then*

$$\ell(t(\Phi)) \dot{\sim} 1_\Lambda^\Phi \tag{2}$$

*Proof.* Let $\succcurlyeq$ satisfy **A5**(LOWER). Set $\Gamma$ to denote $\Lambda \setminus \Phi$. Then, by the representation for bets,

$$1_{\Lambda\setminus\Phi}^\Gamma = 1_\Gamma \underset{\cdot}{\sim} \ell(t(\Gamma)) \quad \text{and} \quad 1_\Lambda^{\Phi\cup\Gamma} = 1_\Lambda \underset{\cdot}{\sim} \ell(t(\Lambda)).$$

So by **A5**(LOWER) we have $1_\Lambda^\Phi \underset{\cdot}{\sim} \ell(t(\Lambda)) - \ell(t(\Gamma)) = \ell(t(\Lambda)) - \ell(t(\Lambda \setminus \Phi))$.

Now assume $\succcurlyeq$ also satisfies **A5**(UPPER). Again, by definition

$$\ell(t(\Lambda)) \dot{\sim} 1_\Lambda$$

Applying the just established result yields $1_\Lambda^\Gamma \underset{\cdot}{\sim} \ell(t(\Lambda)) - \ell(t(\Phi))$. So by **A5**(UPPER) we have

$$\ell(t(\Lambda)) - \ell(t(\Lambda)) - \ell(t(\Phi)) = \ell(t(\Phi)) \dot{\sim} 1_\Lambda^\Phi. \quad \blacksquare$$

Define $\underline{\boldsymbol{f}} = w \mapsto \min\{f(\varphi) \mid w \in t(\varphi)\}$ and $\overline{\boldsymbol{f}} = w \mapsto \max\{f(\varphi) \mid w \in t(\varphi)\}$. Clearly, $\underline{\boldsymbol{f}}, \overline{\boldsymbol{f}} \in \{\!\{f\}\!\}$ and

$$\int \underline{\boldsymbol{f}} \mathrm{d}\ell \leq \int \boldsymbol{f} \mathrm{d}\ell \leq \int \overline{\boldsymbol{f}} \mathrm{d}\ell$$

for any other $\boldsymbol{f} \in \{\!\{f\}\!\}$.

LEMMA 5. *Let* $f, g : \Lambda \to \mathbb{R}_+$ *be co-monotone, set* $h = \alpha f + \beta g$. *Then* $\underline{\boldsymbol{h}} = \alpha \underline{\boldsymbol{f}} + \beta \underline{\boldsymbol{g}}$ *and* $\overline{\boldsymbol{h}} = \alpha \overline{\boldsymbol{f}} + \beta \overline{\boldsymbol{g}}$.

*Proof.* This is immediate from the fact that $\operatorname{argmin}_{t^{-1}(w)} f(\varphi) \cap \operatorname{argmin}_{t^{-1}(w)} g(\varphi) \neq \varnothing$, by co-monotonicity, so for each $w$ there exists a $\Phi \in t^{-1}(w)$ such that $\alpha f(\Phi) + \beta g(\Phi) = \alpha \underline{\boldsymbol{f}}(w) + \beta \underline{\boldsymbol{g}}(w)$, which is clearly less than $h(\Psi)$ for any other $\Psi \in t^{-1}(w)$. By analogy we have this also for $\overline{\boldsymbol{h}}$. $\blacksquare$

For $1_\Lambda^\Phi$, we have that

$$\underline{\mathbf{1}_{\boldsymbol{\Lambda}}^{\boldsymbol{\Phi}}} : w \to \begin{cases} 1 \text{ if } w \in t(\Phi) \setminus t(\Lambda \setminus \Phi), \\ 0 \text{ otherwise,} \end{cases} \qquad \overline{\mathbf{1}_{\boldsymbol{\Lambda}}^{\boldsymbol{\Phi}}} : w \to \begin{cases} 1 \text{ if } w \in t(\Phi), \\ 0 \text{ otherwise.} \end{cases} \tag{3}$$

Let $\succcurlyeq$ satisfy **A1–A5**. Applying Lemma 4 to (3) we have:

$$\int \overline{\mathbf{1}_{\mathbf{\Lambda}}^{\mathbf{\Phi}}}\, \mathrm{d}\ell \;\dot{\sim}\; 1_{\Lambda}^{\Phi} \qquad \text{and} \qquad 1_{\Lambda}^{\Phi} \;\underset{\cdot}{\sim}\; \int \underline{\mathbf{1}_{\mathbf{\Lambda}}^{\mathbf{\Phi}}}\, \mathrm{d}\ell$$

Now, for any $f$ with $\Lambda = \textsc{dom}(f)$, we can find $\{\Gamma_n\}_{n=1}^{m}$ with $\Gamma_{n+1} \subseteq \Gamma_n \subseteq \Lambda$ such that

$$f = \sum_{n=1}^{m} \alpha_n 1_{\Lambda}^{\Gamma_n}$$

for $\alpha_n \in \mathbb{R}_+$. By their nestedness these bets are pairwise co-monotone, we have

$$\begin{aligned}
f &= \sum_{n=1}^{m} \alpha_n 1_{\Lambda}^{\Gamma_n} \\
&\underset{\cdot}{\sim} \sum_{n=1}^{m} \alpha_n \int \underline{\mathbf{1}_{\mathbf{\Lambda}}^{\mathbf{\Gamma_n}}}\, \mathrm{d}\ell && \text{(by } \mathbf{A4} \text{ and Lemma 3)} \\
&= \int \sum_{n=1}^{m} \alpha_n \underline{\mathbf{1}_{\mathbf{\Lambda}}^{\mathbf{\Gamma_n}}}\, \mathrm{d}\ell && \text{(by linearity of Choquet integral)} \\
&= \int \underline{\boldsymbol{f}}\, \mathrm{d}\ell && \text{(by Lemma 5)}
\end{aligned}$$

A repetition of the argument shows also that

$$\int \overline{\boldsymbol{f}}\, \mathrm{d}\ell \;\dot{\sim}\; f.$$

Hence, $f \succ g$ if and only if (by Lemma 2) $\int \underline{\boldsymbol{f}}\, \mathrm{d}\ell > \int \overline{\boldsymbol{g}}\, \mathrm{d}\ell$, which given the construction of $\underline{\boldsymbol{f}}$ and $\overline{\boldsymbol{g}}$, implies exactly the desired representation. ■

## B.1 Maxmin IDEU

Thought of as a criteria on strict preference (i.e., taking $\succ$ as the observable), sparse IDEU preferences are often far from complete. In other words, the representation (SIDEU) is agnostic about how to compare acts when different translations of the acts can be ranked in different ways (i.e., when the interval of constant equivalent bets overlap).

The next criteria we consider is *MaxMin IDEU*, which can be thought of as the cautious completion of this order. Whenever two acts are not strictly ranked by sparse

IDEU, MaxMin selects the act with the higher worst-case consistent translation. Formally:

Definition 1 (MaxMin preferences). *Call* $\succcurlyeq$ *a* Maxmin preference *if there exits some interpretation of uncertainty* $(W, t, \mu)$, *with additive* $\mu$, *such that for all syntactic acts* $f$ *and* $g$,

$$f \succcurlyeq g \quad \Leftrightarrow \quad \min_{\boldsymbol{f} \in \{\!\{f\}\!\}} \int \boldsymbol{f} \, \mathrm{d}\ell \;\geq\; \min_{\boldsymbol{g} \in \{\!\{g\}\!\}} \int \boldsymbol{g} \, \mathrm{d}\ell \tag{MM-IDEU}$$

It is rather immediate that if (SIDEU) ranks two bets then (MM-IDEU) will preserve this ranking. However, unlike sparse IDEU, the later is a true decision criteria as it recounts a complete preference.

Theorem 6. *A relation* $\succcurlyeq$ *is complete and transitive and satisfies* **A2**–**A4** *and* **A5**-(LOWER) *if and only if it is a MaxMin IDEU preference.*

As is apparent by the overlap in their axiomatization, the sparse and maxmin choice criteria are closely related. Indeed, given the same IOU, these two preferences will agree on any strict preference indicated by sparse-IDEU. However, to extend the choice procedure to all acts, as in MaxMin IDEU, clearly requires the preference be complete and transitive. Since a complete relation satisfies **A1**, we can drop this dictate from conditions for MaxMin IDEU.

*Proof.* Let $\succcurlyeq$ be complete and transitive and satisfy **A2**–**A4** and **A5**-(LOWER).

First, we claim that if $f \mathrel{\dot{\sim}} g$ then $f \sim g$. Since the asymmetric component of a relation is necessarily irreflexive, we have $f, g \notin \{h \mid f \succ h\} = \{h \mid g \succ h\}$ (where the equality is the definition of $\dot{\sim}$) or that $f \not\succ g$ and $g \not\succ f$ so by completeness $f \sim g$.

Following the proof of Theorem 1, we have that $f \mathrel{\dot{\sim}} \int \underline{\boldsymbol{f}} \, \mathrm{d}\ell$. which by the above observation implies $f \sim \int \underline{\boldsymbol{f}} \, \mathrm{d}\ell$. This completes the proof as: $f \succcurlyeq g$ if and only if $\int \underline{\boldsymbol{f}} \, \mathrm{d}\ell \succcurlyeq \int \underline{\boldsymbol{g}} \, \mathrm{d}\ell$ if and only if $\int \underline{\boldsymbol{f}} \, \mathrm{d}\ell \geq \int \underline{\boldsymbol{g}} \, \mathrm{d}\ell$, which given the construction of $\underline{\boldsymbol{f}}$ implies exactly the desired representation. ■

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